Score:

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Period (circle one): 1 2 3 4 5 6
Team (circle one): a b c d e f

SM286 – Final Examination– Spring 2011

Approximate all Answers to 2 Decimal Places

1. Radioactive decay is governed by the 1st order differential equation $\frac{dM}{dt} = kM$ with the initial condition $M(0) = M_0$. A scrap of paper taken from the Dead Sea scrolls contains .795 times the original mass of Carbon-14 (^{14}C) that it had when the scrolls were written. If ^{14}C has a half life of 5700 years how old are the scrolls (Note: Use DE methods to solve this problem).

2. The equation for current in a simple CR series circuit is L$\frac{d^2q}{dt^2} + \frac{1}{C}q = V$, where q is charge, L is inductance, C is capacitance, V is the voltage, and t is time.

   a. Solve for charge if the circuit consists of a 5 Henry inductor, .1 farad capacitor, and a 10 volt battery. Assume that the initial charge $q(0) = 0$ coulombs and the initial current $i(0) = 4$ amps (recall that $i = \frac{dq}{dt}$).
   b. Combine the "$\sin/\cos$" part of your answer in part a as a single expression $Asin(\omega t + \phi)$.

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3. Solve the differential equation $x\frac{dy}{dx} - 4y = x^6 e^x$ where $y(1) = 6$.

4. Solve the following system of D.E.'s $\begin{cases} \frac{dx}{dt} = -2x + 2y \\ \frac{dy}{dt} = \quad\; x - 3y \end{cases}$ with initial conditions $\begin{cases} x(0) = 4 \\ y(0) = 5 \end{cases}$.

Using your solution, calculate $x(.2)$ and $y(.2)$.

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5. Using the system of equations in Problem 4:
   a. Use Euler's method to estimate the value of $x(.2)$ using a step size of $\Delta t = .1$.
   b. Why does this answer differ from the answer in Problem 4?
   c. How would you improve this answer?

6. Using matrices, balance the chemical formula:

$$\_\_HClO_4 + \_\_P_4O_{10} \rightarrow \_\_H_3PO_4 + \_\_Cl_2O_7$$

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7. Using the definition of the Maclaurin series: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$:
   a. **Derive** the Maclaurin series for $\frac{1}{1+x}$.
   b. **Using you answer in part a**, derive the Maclaurin series for $\frac{1}{1+x^3}$?
   c. **Using you answer in part a**, derive Maclaurin series for $\frac{1}{1-x^2}$?

8. Solve the differential equation $\frac{d^2x}{dt^2} + t\frac{dx}{dt} = 0,\ x(0) = 2,\ x'(0) = 1.$ (Hint: Let $x = \sum_{n=0}^{\infty} c_n t^n.$) Write out the 1st four non-zero terms of the series.

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9. The temperature in a one dimension rod is governed by the following PDE:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \text{ with boundary and initial conditions } \begin{cases} u(0,t) = x(\pi,t) = 0 \\ u(x,0) = (x-\pi)^2 x^2 \end{cases}.$$

Assume that the eigenvalues for $\boldsymbol{\lambda} \leq 0$ produce trivial solutions. Solve for $u(x,t)$.

10. The equation for a vibrating beam is governed by the PDE $\frac{\partial^2 u}{\partial t^2} = -\alpha^2 \frac{\partial^4 u}{\partial x^4}$ where u is the displacement of the beam and α is a constant that embodies the beam properties. Assume that $u(x, t)$ is separable and use separation of variables techniques to express the PDE as two ordinary differential equations. (DO NOT SOLVE THE RESULTING ODES!)