Homework Solutions

Section 1.1
(10) 1, 2, 13, 19, 27, 29

1) $(1-x)y^{(n)} - 4xy' + 5y = \cos x$

$\Rightarrow$ n^{th} order

$\Rightarrow$ linear, all dependent variable terms (y) are linear

2) $x\frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^4 + y = 0$

$\Rightarrow$ 3RD order

$\Rightarrow$ non-linear due to $\left(\frac{dy}{dx}\right)^4$ term

note: not the same as $\frac{d^4y}{dx^4}$

13) $y = e^{3x}\cos(2x)$

$y' = 3e^{3x}\cos(2x) - 2e^{3x}\sin(2x)$

$y'' = 9e^{3x}\cos(2x) - 6e^{3x}\sin(2x) - 6e^{3x}\sin(2x) - 4e^{3x}\cos(2x)$

$= 5e^{3x}\cos(2x) - 12e^{3x}\sin(2x)$

$y'' = 5e^{3x}\cos(2x) - 12e^{3x}\sin(2x)$

$-6y' = -18e^{3x}\cos(2x) + 12e^{3x}\sin(2x)$

$13y = 13e^{3x}\cos(2x)$

$= 0 + 0 \Rightarrow y'' - 6y' + 13y = 0$ ✓✓

19) $\frac{d}{dt}\ln\left(\frac{2x-1}{x-1}\right) = \frac{x-1}{2x-1}\frac{d}{dt}\left[\frac{2x-1}{x-1}\right]$

Implicit differentiation step

$$= \frac{x-1}{2x-1}\,\frac{(x-1)(2)(x') - (2x-1)(1)(x')}{(x-1)^2}$$

$$= \left(\frac{x-1}{2x-1}\right)\frac{(2x-2-2x+1)}{(x-1)^2}\,x'$$

$$= \frac{\cancel{(x-1)}}{(2x-1)}\,\frac{-1}{\cancel{(x-1)}}\,x'$$

$$= \frac{1}{(1-2x)(x-1)}\,x' = \frac{d}{dt}(t) = 1$$

$$\Rightarrow \boxed{x'' = (1-2x)(x-1)}$$

27) $y = e^{mx}$, $y' = me^{mx}$

$$\Rightarrow me^{mx} + 2e^{mx} = 0$$

$$\Rightarrow \boxed{m = -2}$$

29) $y'' = m^2 e^{mx} \Rightarrow (m^2 - 5m + 6)e^{mx} = 0$

$$\Rightarrow (m-2)(m-3) = 0 \Rightarrow \boxed{m = 2, 3}$$