HOMEWORK SOLUTIONS

Section 1.2

(17) #1, 9, 13, 14, 17, 25, 26

1) $y(0) = 1/(1+ce^0) = -1/3$

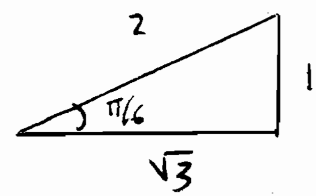
$\Rightarrow 1+c = -3 \Rightarrow c = -4$

$\therefore \boxed{y(x) = 1/(1-4e^{-x})}$

9) $x = C_1\cos(t) + C_2\sin(t)$

$x' = -C_1\sin(t) + C_2\cos(t)$ ← didn't need this

$\cancel{x'' = -C_1\cos(t) - C_2\sin(t)}$



$x(\pi/6) = c_1\cos(\pi/6) + C_2\sin(\pi/6)$

$x(\pi/6) = \frac{\sqrt{3}}{2}C_1 + \frac{1}{2}C_2 = 1/2$

$x'(\pi/6) = -\frac{1}{2}C_2 + \frac{\sqrt{3}}{2}C_2 = 0$

$\Rightarrow \begin{array}{l} \sqrt{3}C_1 + C_2 = 1 \\ -C_1 + \sqrt{3}C_2 = 0 \end{array} \Rightarrow \begin{array}{l} \sqrt{3}C_1 + C_2 = 1 \\ -\sqrt{3}C_1 + 3C_2 = 0 \\ \hline 4C_2 = 1 \Rightarrow C_2 = 1/4 \end{array}$

$\Rightarrow C_1 = \sqrt{3}/4$

$\therefore \boxed{x = \frac{\sqrt{3}}{4}\cos(t) + \frac{1}{4}\sin(t)}$

13) $y = c_1 e^{x} + c_2 e^{-x}$

$y' = c_1 e^{x} - c_2 e^{-x}$

~~$y'' = c_1 e^{x} + c_2 e^{-x}$~~ didn't need this

$\Rightarrow y(-1) = c_1 e^{-1} + c_2 e = 5$

$y'(-1) = c_1 e^{-1} - c_2 e = -5$

$2c_1 e^{-1} = 0 \Rightarrow c_1 = 0 \quad \therefore c_2 = 5/e$

$\therefore y = \frac{5}{e} e^{-x} \Rightarrow \boxed{y = 5e^{-(x+1)}}$

14) $y(0) = c_1 + c_2 = 0$

$y'(0) = c_1 - c_2 = 0 \Rightarrow 2c_1 = 0 \Rightarrow c_1 = 0 \Rightarrow c_2 = 0$

$\boxed{y = 0}$

17) let $f(x,y) = y^{2/3} \Rightarrow$ continuous for all y

$\Rightarrow \partial f/\partial y = \frac{2}{3} y^{-1/3} \Rightarrow$ continuous for $y \neq 0$

Region 1: Half-plane for $y > 0$

Region 2: Half-plane for $y < 0$

25) let $f(x,y) = (y^2 - 9)^{1/2}$

$\Rightarrow \frac{\partial f}{\partial y} = \frac{1}{2}(y^2 - 9)^{-1/2} \Rightarrow$ continuous for $y \neq 3, -3$

$\therefore$ there is a unique solution through $(1,4)$

26) but there is <u>not</u> a unique solution through $(5,3)$