Homework Solutions Section 2.2

(SO) 1, 6, 7, 17, 25

1) $\int dy = \int \sin(5x)\,dx \Rightarrow \boxed{y = -\frac{1}{5}\cos(5x) + C}$

6) $\frac{dy}{dx} = -2xy^2 \Rightarrow \frac{dy}{y^2} = -2x\,dx$

$\Rightarrow \int y^{-2}dy = -2\int x\,dx \Rightarrow -y^{-1} = -x^2 + C$

$\Rightarrow \frac{1}{y} = x^2 + C \Rightarrow \boxed{y = \frac{1}{x^2+C}}$

7) $\frac{dy}{dx} = e^{3x}e^{2y} \Rightarrow \int e^{-2y}dy = \int e^{3x}dx$

$= -\frac{1}{2}e^{-2y} = \frac{1}{3}e^{3x} + C$

$\Rightarrow \boxed{e^{-2y} = -\frac{2}{3}e^{3x} + C} \Rightarrow -2y = \ln(-\frac{2}{3}e^{3x} + C)$

$\Rightarrow \boxed{y = -\frac{1}{2}\ln\left(-\frac{2}{3}e^{3x} + C\right)}$

$-P+1+P$

17) $\frac{dP}{P(1-P)} = dt \Rightarrow \frac{1}{P(1-P)} = \frac{A}{P} + \frac{B}{1-P} \Rightarrow 1 = A(1-P) + BP$

$\Rightarrow P=0 \Rightarrow A=+1,\ P=1 \Rightarrow B=1$

$\Rightarrow \int\left(\frac{+1}{P} + \frac{1}{1-P}\right)dP = \int dt \Rightarrow \ln(P) - \ln(1-P) = t + C$

$\Rightarrow \ln[P/(1-P)] = t + C \Rightarrow P/(1-P) = Ae^t$

$\Rightarrow P = Ae^t - P(Ae^t) \Rightarrow P(1+Ae^t) = Ae^t \Rightarrow \boxed{P = \frac{Ae^t}{1+Ae^t}}$

20) $\frac{dy}{dx} = \frac{(x+2)(y-1)}{(x-3)(y+1)} \Rightarrow \left(\frac{y+1}{y-1}\right)dy = \left(\frac{x+2}{x-3}\right)dx$

$\Rightarrow \frac{(y-1)+2}{(y-1)}\, dy = \frac{(x-3)+5}{(x-3)}\, dx$

$\Rightarrow \left(1 + \frac{2}{y-1}\right)dy = 1 + \frac{5}{(x-3)}\, dx$

$\Rightarrow \boxed{y + 2\ln|y-1| = x + 5\ln|x-3| + C}$

Stop here. can not solve explicitely

25) $x^2 \frac{dy}{dx} = y - xy \Rightarrow x^2 \frac{dy}{dx} = y(1-x)$

$\Rightarrow \frac{dy}{y} = \frac{1-x}{x^2}\, dx = \left(\frac{1}{x^2} - \frac{x}{x^2}\right)dx$

$\Rightarrow \int \frac{dy}{y} = \int\left(\frac{1}{x^2} - \frac{1}{x}\right)dx \Rightarrow \ln y = -\frac{1}{x} - \ln(x) + C$

$\Rightarrow e^{\ln(y)} = e^{-\frac{1}{x} - \ln(x) + C} = e^{-1/x}\, e^{\ln(\frac{1}{x})}\, \underbrace{e^{C}}_{=A}$

$\Rightarrow y = A\frac{1}{x}e^{-1/x}$ Initial Condition $\Rightarrow$ $y(-1) = -1$

$\Rightarrow y = A\left(\frac{1}{-1}\right)e^{-1/-1} = -1 \Rightarrow -Ae = -1 \Rightarrow A = 1/e$

$\therefore \boxed{y = \frac{1}{e}\frac{1}{x}e^{-1/x}}$