Homework Solutions Section 2.3

(60) 3, 5, 10, 23, 27, 28

3) $\frac{dy}{dx} + y = e^{3x}$ $\quad u(x) = e^{\int 1 dx} = e^x$

$\Rightarrow e^x \frac{dy}{dx} + e^x y = e^{4x} \Rightarrow \frac{d}{dx}[e^x y] = e^{4x}$

$\Rightarrow \int d[e^x y] = \int e^{4x} dx \Rightarrow e^x y = \frac{1}{4}e^{4x} + c$

$\Rightarrow \boxed{y = \frac{1}{4}e^{4x} + ce^{-x}}$

- general solution exists for all x (i.e. $x \in (-\infty, \infty)$)
- ce^{-x} is transient (i.e. $ce^{-x} \to 0$ as $x \to \infty$)

5) $\frac{dy}{dx} + 3x^2 y = x^2$ $\Rightarrow u(x) = e^{\int 3x^2 dx} = e^{x^3}$

$\Rightarrow e^{x^3}\frac{dy}{dx} + 3x^2 e^{x^3} = x^2 e^{x^3}$

$\Rightarrow \int d[e^{x^3} y] = \int x^2 e^{x^3} dx$

$\Rightarrow e^{x^3} y = \frac{1}{3}e^{x^3} + c \Rightarrow \boxed{y = \frac{1}{3} + \underbrace{ce^{-x^3}}_{\text{transient}}}$ $\quad x \in (-\infty, \infty)$

10) $\frac{dy}{dx} + \frac{2}{x}y = \frac{3}{x}$ $\Rightarrow \mu = e^{\int \frac{2}{x}dx} = e^{2\ln x} = e^{\ln(x^2)} = x^2$

$\Rightarrow \int d[x^2 y] = \int 3x\,dx \Rightarrow x^2 y = \frac{3}{2}x^2 + c$

$\Rightarrow \boxed{y = \frac{3}{2} + cx^{-2}} \Rightarrow x \neq 0$

transient

23) $\frac{dy}{dx} + \frac{(3x+1)}{x}y = \frac{e^{-3x}}{x} =$

$\Rightarrow \mu = e^{\int(3+\frac{1}{x})dx} = e^{3x+\ln(x)} = e^{3x}e^{\ln(x)} = xe^{3x}$

$\Rightarrow d[xe^{3x}y] = 1dx \Rightarrow xe^{3x}y = x+c$

$\Rightarrow \boxed{y = e^{-3x} + cx^{-1}e^{-3x}} \Rightarrow x \neq 0$

entire solution is transient

27) $L\frac{di}{dt} + Ri = E \qquad i(0) = i_0$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}i = \frac{E}{L} \qquad \mu = e^{\frac{R}{L}t}$$

$$\Rightarrow \int d\left[e^{\frac{R}{L}t} i\right] = \int \frac{E}{L} e^{\frac{R}{L}t}\, dt$$

$$\Rightarrow e^{\frac{R}{L}t} i = \frac{E}{R} e^{\frac{R}{L}t} + C$$

$$\Rightarrow i = \frac{E}{R} + Ce^{-\frac{R}{L}t}$$

$$\Rightarrow i(0) = \frac{E}{R} + C = 0 \Rightarrow C = -E/R$$

$$\Rightarrow \boxed{i = \frac{E}{L}\left(1 - \underbrace{e^{-\frac{R}{L}t}}_{\text{transient}}\right)} \qquad t \in (-\infty, \infty)$$

28) $\frac{dT}{dt} - kT = -kT_m \qquad T(0) = T_0$

$\Rightarrow \quad \mu(t) = e^{-kt}$

$\Rightarrow \int d\left[e^{-kt}T\right] = \int -kT_m e^{-kt}\,dt$

$\Rightarrow \quad e^{-kt}T = T_m e^{-kt} + C$

$\Rightarrow \quad T = T_m + Ce^{kt} \Rightarrow T(0) = T_m + C = T_0$

$\Rightarrow \quad C = T_0 - T_m$

$\Rightarrow \quad \boxed{T = T_m + (T_0 - T_m)e^{kt}} \qquad t \in (0,\infty)$

transient for $k<0$