Homework Solutions

Section 4.1a
Higher Order DEs
(128) 3, 5, 7, 9, 10

3) $y = c_1 x + c_2 x \ln(x)$

$y(1) = c_1(1) + c_2(1)\underbrace{\ln(1)}_{\to 0} = 3 \Rightarrow \underline{c_1 = 3}$

$\Rightarrow y = 3x + c_2 x \ln(x) \Rightarrow y' = 3 + c_2 \ln(x) + c_2 \underbrace{\left(\frac{x}{x}\right)}_{\to 1}$

$\Rightarrow y'(1) = 3 + c_2 \underbrace{\ln(1)}_{\to 0} + c_2 = -1 \Rightarrow \underline{c_2 = -4}$

$\therefore \boxed{y = 3 - 4x\ln(x)}$

5) $y = c_1 + c_2 x^2 \Rightarrow y(0) = c_1 = 0$

$\Rightarrow y = c_2 x^2 \Rightarrow y' = 2c_2 x \Rightarrow y'(0) = 2c_2(0) = 1$

$\Rightarrow 0 = 1$ inconsistant $\Rightarrow$ <u>no solution</u>

Does not violate Theorem 4.1.1 because for the D.E. $xy'' - y' = 0$, the coefficient $a_2(x)$ [i.e. $a_2(x) = x$] is 0 when x is 0. Since our initial conditions are stated for $x = 0$, we have <u>no</u> assurance of a solution

7) $x(0) = c_1 \underbrace{\cos(0)}_{\to 1} + c_2 \underbrace{\sin(0)}_{\to 0} = x_0 \Rightarrow c_1 = x_0$

$\therefore x(t) = x_0 \cos(\omega t) + c_2 \sin(\omega t)$

$\Rightarrow x'(t) = -\omega x_0 \sin(\omega t) + c_2 \omega \cos(\omega t)$

$\Rightarrow x'(0) = c_2 \omega = x_1 \Rightarrow c_2 = x_1/\omega$

$\Rightarrow \boxed{x(t) = x_0 \cos(\omega t) + (x_1/\omega)\sin(\omega t)}$

9) $(x-2)y'' + 3y = x$

$\Downarrow$

so long as $x \neq 2$ there is a unique solution $\Rightarrow$ $x = (-\infty, 2)$

interval "centered" on zero

10) $y'' + \tan(x)y = e^x$

$\Uparrow$

this term is discontinuous at $\ldots -3\pi/2, -\pi/2, \pi/2, 3\pi/2, \ldots$

$\therefore \quad x \in (-\pi/2, \pi/2)$