Homework Solutions

Section 4.1b
Higher ORDER D.E.s
(129) 15, 18, 23, 25, 31

15) Since $f_3(x) = 4f_1(x) - 3f_2(x)$, the three solutions are not linearly independent!

**OR** ⇒ By using the Wronskian we have

$$W = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ f_1'(x) & f_2'(x) & f_3'(x) \\ f_1''(x) & f_2''(x) & f_3''(x) \end{vmatrix} = \begin{vmatrix} x & x^2 & 4x-3x^2 \\ 1 & 2x & 4-6x \\ 0 & 2 & -6 \end{vmatrix}$$

$$= x\begin{vmatrix} 2x & 4-6x \\ 2 & -6 \end{vmatrix} - 1\begin{vmatrix} x^2 & 4x-3x^2 \\ 2 & -6 \end{vmatrix} + \underbrace{0\begin{vmatrix} - & - \\ - & - \end{vmatrix}}_{\to 0}$$

$$= x(\cancel{-12x} - 8 + \cancel{12x}) - (\cancel{-6x^2} - 8x + \cancel{6x^2})$$

$$= -8x + 8x = 0$$

∴ <u>not linearly independent</u>

18) $$W = \begin{vmatrix} \cos(2x) & 1 & \cos^2(x) \\ -2\sin(2x) & 0 & -2\cos(x)\sin(x) \\ -4\cos(2x) & 0 & +2\sin^2(x) - 2\cos^2(x) \end{vmatrix}$$

$$\therefore \; -1\begin{vmatrix} -2\sin(2x) & -2\cos(x)\sin(x) \\ -4\cos(2x) & 2\sin^2(x) - 2\cos^2(x) \end{vmatrix} + \underbrace{0\begin{vmatrix} - & - \\ - & - \end{vmatrix}}_{\to 0} - \underbrace{0\begin{vmatrix} - & - \\ - & - \end{vmatrix}}_{\to 0}$$

↙ TRIG ID's

$$= -1\begin{vmatrix} -2\sin(2x) & -\sin(2x) \\ -4\cos(2x) & -2\cos(2x) \end{vmatrix} =$$

$$\boxed{\therefore \text{ not linearly independent}}$$

$$= 4\sin(2x)\cos(2x) - 4\sin(2x)\cos(2x) = 0$$

23) $\therefore$ $\left.\begin{array}{l} y = e^{-3x} \\ y' = -3e^{-3x} \\ y'' = 9e^{-3x} \end{array}\right\}$ $\therefore\ y'' - y' - 12y = (9-(-3)-12)\,e^{-3x} = 0$ ✓✓

$\left.\begin{array}{l} y = e^{4x} \\ y' = 4e^{4x} \\ y'' = 16e^{4x} \end{array}\right\}$ $y'' - y' - 12y = (16-4-12)e^{4x} = 0$ ✓✓

Solutions work $\Rightarrow$ check for indendendence

$$W = \begin{vmatrix} e^{-3x} & e^{4x} \\ -3e^{-3x} & 4e^{4x} \end{vmatrix} = 4e^{x} - (-3e^{x}) = 7e^{x} \neq 0 \text{ ✓✓}$$

$$\therefore\ \boxed{y = C_1 e^{-3x} + C_2 e^{4x}}$$

25) $\left\{\begin{array}{l} y = e^{x}\cos(2x) \\ y' = e^{x}\cos(2x) - 2e^{x}\sin(2x) \\ y'' = e^{x}\cos(2x) - 2e^{x}\sin(2x) - 2e^{x}\sin(2x) - 4e^{x}\cos(2x) \end{array}\right.$

$\quad = -4e^{x}\sin(2x) - 3e^{x}\cos(2x)$

$y'' - 2y' + 5y = (-3-2+5)e^{x}\cos(2x) + (-4+4)e^{x}\sin(2x)$

$= 0$ ✓✓ $1^{ST}$ solution works

$\left\{\begin{array}{l} y = e^{x}\sin(2x) \\ y' = e^{x}\sin(2x) + 2e^{x}\cos(2x) \\ y'' = e^{x}\sin(2x) + 2e^{x}\cos(2x) + 2e^{x}\cos(2x) - 4e^{x}\sin(2x) \end{array}\right.$

$\quad = -3e^{x}\sin(2x) + 4e^{x}\cos(2x)$

$y'' - 2y' + 5y = (-3-2+5)e^{x}\sin(2x) + (4-4)e^{x}\cos(2x) = 0$ ✓✓

2nd solution works.

25) continued $\Rightarrow$ check for linear independence

$$W = \begin{vmatrix} e^{x}\cos(2x) & e^{x}\sin(2x) \\ e^{x}\cos(2x) - 2e^{x}\sin(2x) & e^{x}\sin(2x) + 2e^{x}\cos(2x) \end{vmatrix}$$

$$= \cancel{e^{2x}\sin(2x)\cos(2x)} + 2e^{2x}\cos^{2}(2x)$$

$$- \left(\cancel{e^{2x}\sin(2x)\cos(2x)} - 2e^{x}\sin^{2}(2x)\right)$$

$$= 2e^{2x}\left(\sin^{2}(2x) + \cos^{2}(2x)\right) = 2e^{2x} \neq 0$$

$\therefore$ linearly indedendent

31) $y = c_1 e^{2x} + c_2 e^{5x} + 6e^{x}$

$y' = 2c_1 e^{2x} + 5c_2 e^{5x} + 6e^{x}$

$y'' = 4c_1 e^{2x} + 25c_2 e^{5x} + 6e^{x}$

$\Rightarrow y'' - 7y' + 10y$

$= 4c_1 e^{2x} + 25c_2 e^{5x} + 6e^{x} - 14c_1 e^{2x} - 35c_2 e^{5x} - 42e^{x}$
$+ 10c_1 e^{2x} + 10c_2 e^{5x} + 60e^{x}$

$= \underbrace{(4c_1 - \cancel{14c_1} + 10c_1)}_{\Rightarrow 0} e^{2x} + \overbrace{(25c_2 \cancel{-} 35c_2 + 10c_2)}^{0} e^{5x}$
$+ (6 - 42 + 60)e^{x} = 24e^{x}$ ✓✓