

# HOMEWORK SOLUTIONS

Section 4.3  
(139) 3, 5, 15, 31, 35

$$3) (D^2 - D - 6)y = 0 \Rightarrow (D-3)(D+2)y = 0$$

$$\Rightarrow D=3, D=-2 \Rightarrow \boxed{y = c_1 e^{3x} - c_2 e^{-2x}}$$

$$5) (D^2 + 8D + 16)y = 0 \Rightarrow (D+4)(D+4)y = 0$$

$$\Rightarrow D = -4, -4 \Rightarrow \boxed{y = c_1 e^{-4x} + c_2 x e^{-4x}}$$

$$15) (D^3 - 4D^2 - 5D)y = D(D^2 - 4D - 5)y = 0$$

$$\Rightarrow D(D-5)(D+1)y = 0 \Rightarrow D=0, 5, -1$$

$$\therefore y = c_1 \overset{\rightarrow}{e^0} + c_2 e^{-x} + c_3 e^{5x}$$

$$\Rightarrow \boxed{y = c_1 + c_2 e^{-x} + c_3 e^{5x}}$$

Note: ON PROBLEMS 3, 5, 15 I WAS NOT GIVEN AN INDEPENDENT VARIABLE. THEREFORE I ASSUME IT TO BE  $x$ . YOU CAN CHOOSE WHATEVER YOU WISH. FOR EXAMPLE:

$$y = c_1 e^{3t} - c_2 e^{-2t}$$

WOULD HAVE ALSO BEEN CORRECT FOR #1

$$3) (D^2 - 4D - 5)y = 0 \quad y(1) = 0, y'(1) = 2$$

t is the independent variable

$$\Rightarrow (D-5)(D+1)y = 0 \Rightarrow y = C_1 e^{5t} + C_2 e^{-t}$$
$$y' = 5C_1 e^{5t} - C_2 e^{-t}$$

$$y(1) = C_1 e^5 + C_2 e^{-1} = 0$$
$$+ y'(1) = 5C_1 e^5 - C_2 e^{-1} = 2$$

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$$6C_1 e^5 = 2 \Rightarrow C_1 = \frac{1}{3} e^{-5}$$
$$\Rightarrow 5 \left(\frac{1}{3}\right) e^5 (e^5) - C_2 e^{-1} = 2$$
$$\Rightarrow \frac{5}{3} - 2 = C_2 e^{-1} \Rightarrow C_2 e^{-1} = -\frac{1}{3}$$
$$\Rightarrow C_2 = -\frac{1}{3} e^1$$

$$\therefore y = \frac{1}{3} e^{-5} e^{5t} - \frac{1}{3} e^1 e^{-t}$$

$$\Rightarrow \boxed{y = \frac{1}{3} e^{5(t-1)} - \frac{1}{3} e^{-(t-1)}}$$

$$35) (D^3 + 12D^2 + 36D)y = 0$$

$$D(D+6)(D+6)y = 0 \Rightarrow D = 0, -6, -6$$

$$y = C_1 + C_2 e^{-6t} + C_3 t e^{-6t}$$

$$y' = -6C_2 e^{-6t} + C_3 e^{-6t} - 6C_3 t e^{-6t}$$

$$= (C_3 - 6C_2) e^{-6t} - 6C_3 t e^{-6t}$$

$$y'' = (-6C_3 + 36C_2) e^{-6t} - 6C_3 e^{-6t} + 36C_3 t e^{-6t}$$

$$= (-12C_3 + 36C_2) e^{-6t} + 36C_3 t e^{-6t}$$

Initial conditions

$$\begin{cases} y(0) = C_1 + C_2 = 0 \\ y'(0) = C_3 - 6C_2 = 1 \\ y''(0) = -12C_3 + 36C_2 = -7 \end{cases}$$

$$\Rightarrow 6C_3 - 36C_2 = 6$$

$$-12C_3 + 36C_2 = -7$$

$$-6C_3 = -1 \Rightarrow \underline{\underline{C_3 = \frac{1}{6}}}, \quad \frac{1}{6} - 6C_2 = 1 \Rightarrow \underline{\underline{C_2 = -\frac{5}{36}}}$$

$$\underline{\underline{C_1 = \frac{5}{36}}}$$

$$\therefore \boxed{y = \frac{5}{36} - \frac{5}{36} e^{-6t} + \frac{1}{6} t e^{-6t}}$$