Homework Solutions

Section 4.3b

(139) 9, 11, 25, 29, 43-48

9) $(D^2+9)y=0 \Rightarrow D^2=-9 \Rightarrow D=\pm 3i$

$\therefore \boxed{y = c_1 \sin(3x) + c_2 \cos(3x)}$

11) $(D^2-4D+5)y=0$

$\Rightarrow D = \dfrac{4 \pm \sqrt{16-20}}{2} = \dfrac{4 \pm \sqrt{-4}}{2} = \dfrac{4 \pm 2i}{2}$

$= 2 \pm i$

$\therefore \boxed{y = c_1 e^{2x} \sin(x) + c_2 e^{2x} \cos(x)}$

25) $(16D^4 + 24D^2 + 9)y = 0 \Rightarrow (4D^2+3)(4D^2+3)y=0$

$\Rightarrow 4D^2 = -3 \Rightarrow D^2 = -3/4 \Rightarrow D = \pm\frac{\sqrt{3}}{2}i, \pm\frac{\sqrt{3}}{2}i$

repeating root

$\therefore \underline{y = c_1 \sin\left(\frac{\sqrt{3}}{2}x\right) + c_2 \cos\left(\frac{\sqrt{3}}{2}x\right) + c_3 x \sin\left(\frac{\sqrt{3}}{2}x\right) + c_4 x \cos\left(\frac{\sqrt{3}}{2}x\right)}$

29) $(D^2+16)y=0 \qquad y(0)=2\ ,\ y'(0)=-2$

$D=\pm 4i \Rightarrow \quad y=c_1\sin(4x)+c_2\cos(4x)$

$\therefore\ y(0)=c_2=2 \Rightarrow \quad y=c_1\sin(4x)+2\cos(4x)$

$\Rightarrow\ y'=4c_1\cos(4x)-8\sin(4x)$

$\Rightarrow\ y'(0)=4c_1=-2 \Rightarrow\ c_1=-\frac{1}{2}$

$\Rightarrow \boxed{y=-\frac{1}{2}\sin(4x)+2\cos(4x)}$

42)a) Solve D.E. $\Rightarrow (D^2-3D-4)y=0 \Rightarrow (D-4)(D+1)y=0$

$\Rightarrow\ y=c_1e^{4x}-c_2e^{-x}$

$\lim_{x\to\infty} y = \lim_{x\to\infty}(c_1e^{4x}-\underbrace{\cancel{c_2e^{-x}}}_{\to 0}) = \pm\infty$ (depending on c_1)

$\lim_{x\to-\infty} y = \lim_{x\to-\infty}(\underbrace{\cancel{c_1e^{4x}}}_{\to 0}-c_2e^{-x}) = \pm\infty$ (depending on c_2)

$\therefore$ solution is 44a.

42) b) $y''+4y=0 \Rightarrow (D^2+4)y=0 \Rightarrow D=\pm 2i$

$\therefore\ y = c_1 \sin 2x + c_2 \cos 2x$

↑ pure sinusoidal with 2 cycles between 0 & 2π $\Rightarrow$ 48b

c) $(D^2+2D+1)y=0 \Rightarrow (D+1)^2=0 \Rightarrow D=-1,-1$

$\Rightarrow y = c_1 e^{-x} + c_2 x e^{-x}$

$$\left.\begin{aligned} &\lim_{x\to\infty}(c_1e^{-x}+c_2xe^{-x})=0 \\ &\lim_{x\to-\infty}(c_1e^{-x}+c_2xe^{-x})=\infty \end{aligned}\right\} \Rightarrow \text{46c}$$

d) $y''+y=0 \Rightarrow (D^2+1)y=0 \Rightarrow D=\pm i$

$\Rightarrow y = c_1 \sin(x) + c_2 \cos(x)$

$\Rightarrow$ like 'b', but only one cycle from $(0, 2\pi)$ 47d

e) $(D^2+2D+2)y=0 \Rightarrow D = \dfrac{-2\pm\sqrt{4-8}}{2}$

$\Rightarrow \dfrac{-2\pm\sqrt{-4}}{2} = \dfrac{-2\pm 2i}{2} = -1\pm i$

$\therefore\ y = c_1 e^{-x}\sin(x) + c_2 e^{-x}\cos(x)$

$\Rightarrow$ decaying sinusoidal 45e

42) f) $y'' - 3y' + 2y = 0 \Rightarrow (D^2 - 3D + 2)y = 0$

$\Rightarrow (D + 2)(D + 1)y = 0 \Rightarrow D = +2, +1$

$\Rightarrow y = c_1 e^{+2x} + c_2 e^{+x}$ 43f