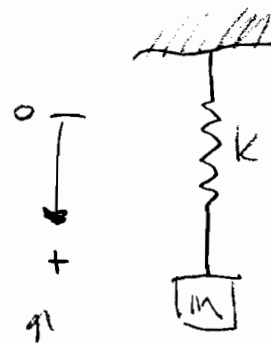


Homework Solutions

Section 5.1b

(194) 9, 11

9) $M = 8 \text{ lbs} / 32 \overset{g}{=} = \frac{1}{4}$
 $k = 1 \text{ lbs/st}$
 $X(0) = \frac{1}{2} \text{ st}$ (6 inches down)
 $X'(0) = \frac{3}{2} \text{ st/sec}$ (downward)



$$\frac{1}{4}x'' + x = 0 \Rightarrow x'' + 4x = 0$$

$$\Rightarrow (D^2 + 4) = 0 \Rightarrow D = \pm 2i$$

$$x = c_1 \sin(2t) + c_2 \cos(2t)$$

$$x(0) = c_2 = \frac{1}{2}$$

$$x = c_1 \sin(2t) + \frac{1}{2} \cos(2t)$$

$$x' = 2c_1 \cos(2t) - \sin(2t)$$

$$x'(0) = 2c_1 = \frac{3}{2} \Rightarrow c_1 = \frac{3}{4}$$

$$\therefore \boxed{x = \frac{3}{4} \sin(2t) + \frac{1}{2} \cos(2t)}$$

$$\Rightarrow A = \left(\left(\frac{3}{4}\right)^2 + \left(\frac{1}{2}\right)^2 \right)^{1/2} = \left(\frac{9}{16} + \frac{1}{4} \right)^{1/2} = \left(\frac{13}{16} \right)^{1/2} = \frac{\sqrt{13}}{4}$$

$$\phi = \tan^{-1} \left(\frac{C \cos}{C \sin} \right) = \tan^{-1} \left(\left(\frac{1}{2}\right) \left(\frac{4}{3}\right) \right) = \tan^{-1} \left(\frac{2}{3} \right)$$

$$= .588$$

$$\Rightarrow \boxed{x = \frac{\sqrt{13}}{4} \sin(2t + .588)}$$

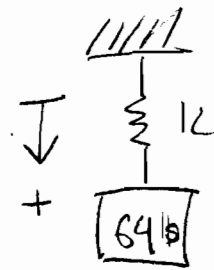
11) Find k

$$\Rightarrow F = kx \Rightarrow 64 = k(0.32 \text{ ft})$$

$$\Rightarrow k = 200 \text{ lbs/ft}$$

$$\Rightarrow m = 64/32 = 2 \quad (F = mg)$$

↑ gravity



up = -
down = +

a) $m x'' + kx = 0 \Rightarrow 2 x'' + 200x = 0$

$$\Rightarrow x'' + 100x = 0$$

$$\Rightarrow x = c_1 \sin(10t) + c_2 \cos(10t)$$

$$\Rightarrow x(0) = c_2 = -\frac{2}{3} \text{ ft} \quad (8'' \text{ up from equilibrium})$$

$$\Rightarrow x = c_1 \sin(10t) - \frac{2}{3} \cos(10t)$$

$$\Rightarrow x' = 10c_1 \cos(10t) - \frac{20}{3} \sin(10t)$$

$$\Rightarrow x'(0) = 10c_1 = 5 \text{ ft/sec} \quad (\text{downward velocity})$$

$$\Rightarrow c_1 = \frac{1}{2}$$

$$\therefore x = \frac{1}{2} \sin(10t) - \frac{2}{3} \cos(10t) = \frac{5}{6} \sin(10t - 92.7^\circ)$$

$$\phi = \tan^{-1} \left(\frac{c \cos}{c \sin} \right)$$

b) $A = \left(\left(\frac{1}{2}\right)^2 + \left(\frac{2}{3}\right)^2 \right)^{1/2} = \left(\frac{1}{4} + \frac{4}{9} \right)^{1/2} = \left(\frac{9}{36} + \frac{16}{36} \right)^{1/2} = \frac{5}{6}$

$$A = \frac{5}{6} \text{ ft}$$

$$P = \frac{1}{f} = \frac{2\pi}{10} = \frac{\pi}{5} \text{ sec/cycle}$$

c) $f = 10/2\pi \text{ cycles/sec}$

after 3π seconds $(3\pi) \left(\frac{10}{2\pi} \right) = 15 \text{ cycles completed}$

$$A) \quad \frac{5}{6} \sin(10t - .927) = 0 \Rightarrow \sin(10t - .927) = 0$$

$$\Rightarrow 10t - .927 = n\pi \Rightarrow \boxed{t = \frac{n\pi + .927}{10}}$$

$n=0 \Rightarrow$ 1st time through downward

$n=1 \Rightarrow$ 1st " " upward

$n=2 \Rightarrow$ 2nd " " downward

$$t = (2\pi + .927)/10 \Rightarrow \boxed{t = .721 \text{ sec}}$$

e) At extremes when $x'(t) = 0$

$$\Rightarrow \frac{25}{3} \cos(10t - .927) = 0$$

$$\Rightarrow 10t - .927 = \frac{(2n+1)\pi}{2} \quad \text{ie. } \cos(\theta) = 0 \text{ for } \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

$$\Rightarrow \boxed{t = \frac{(2n+1)\pi}{20} + .0927}$$

$$e) \quad x(3) = \frac{5}{6} \sin(30 - .927) = -.597 \text{ ft}$$

$$f) \quad x'(3) = \frac{25}{3} \cos(30 - .927) = -5.81 \text{ ft/sec}$$

$$g) \quad x'' = -\frac{250}{3} \sin(10t - .927)$$

$$x''(3) = 59.7 \text{ ft/sec}^2$$

$$h) \quad x'(.721) = \frac{25}{3} \cos(.721 - .927) = \pm 8.33 \text{ ft/sec}$$


\uparrow can be up or down

$$f) \frac{5}{6} \sin(10t - .927) = 5/12 \quad (5'')$$

$$\Rightarrow 10t - .927 = \sin^{-1}(1/2) = .526 \text{ \& } \pi - .526 = 2.614$$

$$\Rightarrow t = .145 + \frac{n\pi}{5} \quad \& \quad t = .355 + \frac{n\pi}{5}$$

↑
period



$$k) x' = \frac{25}{3} \cos(10t - .927) < 0$$