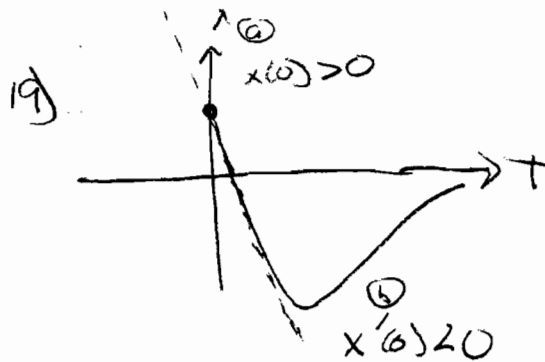
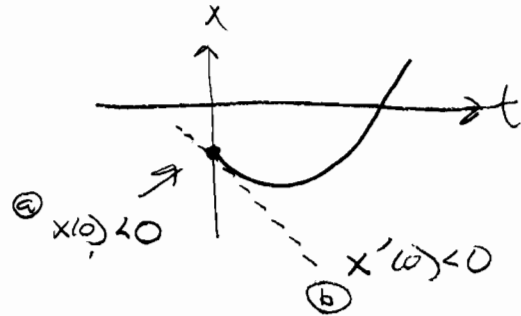
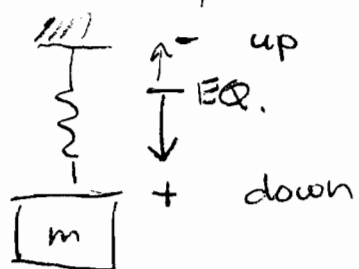


HOMEWORK SOLUTIONS

Section 5.1.2
(194) 17, 19, 21, 23, 25

- 17) a) initial displacement is above the equilibrium position
b) initial velocity is upward



- (a) below (+)
(b) upward (-)

21) $W = 4 \text{ lbs} \therefore m = 4/32 = 1/8 \text{ lbs-m}$
 $k = 2 \text{ lbs/ft}$
 $b = 1$
 $x(0) = -1 \text{ ft (above)}$
 $x'(0) = +8 \text{ ft/sec (downward)}$

$$m x'' + b x' + k x = 0 \Rightarrow \frac{1}{8} x'' + x' + 2x = 0$$

$$\Rightarrow x'' + 8x' + 16x = 0 \Rightarrow (D^2 + 8D + 16)x = 0$$

$$\Rightarrow (D+4)^2 x = 0 \Rightarrow D = -4, -4$$

i. $x = C_1 e^{-4t} + C_2 t e^{-4t}$
 $\Rightarrow x(0) = C_1 = -1 \Rightarrow x = -e^{-4t} + C_2 t e^{-4t}$
 $\Rightarrow x' = 4e^{-4t} + C_2 e^{-4t} - 4C_2 t e^{-4t}$

$$x'(0) = 4 + C_2 = 8 \Rightarrow C_2 = 4$$

$$x = -e^{-4t} + 4te^{-4t}$$

$x=0$ at Equilibrium:

$$\begin{aligned} -e^{-4t} + 4te^{-4t} &= 0 \Rightarrow 4te^{-4t} = e^{-4t} \\ \Rightarrow 4t &= 1 \Rightarrow t = \frac{1}{4} \text{ sec} \end{aligned}$$

$x'=0$ (extreme displacement)

$$\begin{aligned} x' &= 4e^{-4t} + 4e^{-4t} - 16te^{-4t} = 0 \\ \Rightarrow 16te^{-4t} &= 8e^{-4t} \Rightarrow t = \frac{1}{2} \text{ sec} \end{aligned}$$

$$x\left(\frac{1}{2}\right) = -e^{-2} + 4\left(\frac{1}{2}\right)e^{-2} = e^{-2} \approx 0.135 \text{ ft}$$

23)

$$m = 1 \text{ kg}$$

$$k = 16 \text{ N/m}$$

$$b = 10$$

$$m x'' + b x' + k x = 0$$

$$x'' + 10x' + 16x = 0$$

$$(D^2 + 10D + 16)x = 0$$

$$(D + 8)(D + 2)x = 0$$

$$D = -8, D = -2$$

$$\therefore x = c_1 e^{-8t} + c_2 e^{-2t}, \quad x' = -8c_1 e^{-8t} - 2c_2 e^{-2t}$$

a) $x(0) = 1$ (1 m. below eq)

$x'(0) = 0$ (at rest)

$$x(0) = c_1 + c_2 = 1 \Rightarrow$$

$$x'(0) = -8c_1 - 2c_2 = 0$$

$$c_1 + c_2 = 1$$

$$-4c_1 - c_2 = 0$$

$$-3c_1 = 1 \Rightarrow \boxed{c_1 = -1/3}$$

$$\Rightarrow \boxed{c_2 = 4/3}$$

$$\therefore \boxed{x = -\frac{1}{3} e^{-8t} + \frac{4}{3} e^{-2t}}$$

b) now use: $x(0) = 1 \text{ m}$
 $x'(0) = -12 \text{ m/s (up)}$

$$\begin{aligned} x(0) = C_1 + C_2 = 1 &\Rightarrow 2C_1 + 2C_2 = 2 \\ x'(0) = -8C_1 - 2C_2 = -12 &\Rightarrow -8C_1 - 2C_2 = -12 \end{aligned}$$

$$-6C_1 = -10 \Rightarrow C_1 = +\frac{10}{6} = +\frac{5}{3}$$

$$C_2 = -2/3$$

$$\therefore \boxed{x = \frac{5}{3}e^{-8t} - \frac{2}{3}e^{-2t}}$$

25) 2 lbs stretches spring 1 ft

$$\begin{aligned} \therefore F = kx &\Rightarrow 2 \text{ lbs} = k(1 \text{ ft}) \\ \Rightarrow k &= 2 \text{ lbs/ft} \end{aligned}$$

Now

$$W = 3.2 \text{ lbs} \Rightarrow m = 3.2/32 = .1 \text{ lbs}\cdot\text{m}$$

$$k = 2 \text{ lbs/ft}$$

$$b = .4$$

$$x(0) = -1 \text{ (1 ft above)}$$

$$x'(0) = 0 \text{ (at rest)}$$

$$mx'' + bx' + kx = 0 \Rightarrow .1x'' + .4x' + 2x = 0$$

$$\Rightarrow x'' + 4x' + 20x = 0 \Rightarrow (D^2 + 4D + 20)x = 0$$

$$D = \frac{-4 \pm \sqrt{16 - 80}}{2} = \frac{-4 \pm 8i}{2} = -2 \pm 4i$$

$$\therefore x = c_1 e^{-2t} \sin(4t) + c_2 e^{-2t} \cos(4t)$$

$$x(0) = c_2 = -1$$

$$\therefore x = c_1 e^{-2t} \sin(4t) - e^{-2t} \cos(4t)$$

$$x' = -2c_1 e^{-2t} \sin(4t) + 4c_1 e^{-2t} \cos(4t) + 2e^{-2t} \cos(4t) + 4e^{-2t} \sin(4t)$$

$$\Rightarrow x'(0) = 4c_1 + 2 = 0 \Rightarrow c_1 = -\frac{1}{2}$$

$$\therefore x = e^{-2t} \left(-\frac{1}{2} \sin(4t) - \cos(4t) \right)$$

combine these

$$b) A = \left[\left(-\frac{1}{2}\right)^2 + (-1)^2 \right]^{\frac{1}{2}} = \left[\frac{5}{4} \right]^{\frac{1}{2}} = \frac{\sqrt{5}}{2}$$

$$\phi = \tan^{-1} \left(\frac{C \cos}{C \sin} \right) = \tan^{-1} \left(\frac{-1 \cdot 1/2}{-1} \right) = \tan^{-1}(2)$$

$$= 1.107 + \pi = 4.249$$

ADD π SINCE $C \sin < 0$

$$\therefore x = \frac{\sqrt{5}}{2} e^{-2t} \sin(4t + 4.249)$$

$$c) \frac{\sqrt{5}}{2} e^{-2t} \sin(4t + 4.249) = 0$$

$$\Rightarrow \sin(4t + 4.249) = 0$$

$$\Rightarrow 4t + 4.249 = n\pi$$

$$\Rightarrow t = \left(\frac{n\pi - 4.249}{4} \right)$$

$\Rightarrow t > 0$ for 1st time when $n=2$

$$\therefore t = \left(\frac{2\pi - 4.249}{4} \right) = 0.509 \text{ sec}$$

(passes through eq. downward for 1st time)

\Rightarrow passes through eq. upward for 1st time when $n=3$

$$t = \left(\frac{3\pi - 4.249}{4} \right) = \boxed{1.294 \text{ sec}}$$