Homework Solutions

Appendix II
(A18) #47, 49, 53

47) $\left|\begin{bmatrix} -1-\lambda & 2 \\ -7 & 8-\lambda \end{bmatrix}\right| = 0 \Rightarrow (-1-\lambda)(8-\lambda) - (2)(-7) = 0$

$\Rightarrow -8 + \lambda - 8\lambda + \lambda^2 + 14 = 0$

$\Rightarrow \lambda^2 - 7\lambda + 6 = 0$

$\Rightarrow (\lambda - 1)(\lambda - 6) = 0 \Rightarrow \lambda = 1, 6$

<u>for $\lambda = 1$</u>

$$\begin{bmatrix} -1-1 & 2 \\ -7 & 8-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} -2 & 2 \\ -7 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$\Rightarrow -2x_1 + 2x_2 = 0 \Rightarrow x_1 = x_2$, let $x_1 = 1 \Rightarrow x_2 = 1$

$\therefore \boxed{\lambda_1 = 1 \quad \vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

<u>for $\lambda = 6$</u>

$$\begin{bmatrix} -7 & 2 \\ -7 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow -7x_1 + 2x_2 = 0$$

$\Rightarrow 2x_2 = 7x_1$

$\Rightarrow$ let $x_1 = 2 \Rightarrow x_2 = 7$

$\therefore \boxed{\lambda_1 = 6 \quad \vec{v}_2 = \begin{bmatrix} 2 \\ 7 \end{bmatrix}}$

49) $\left|\begin{bmatrix} -8-\lambda & -1 \\ 16 & 0-\lambda \end{bmatrix}\right| = 0 \Rightarrow (-8-\lambda)(-\lambda) - (-1)(16)$

$\Rightarrow 8\lambda + \lambda^2 + 16 = 0 \Rightarrow (\lambda+4)^2 = 0$

$\Rightarrow \lambda = -4, -4$

$\Rightarrow \begin{bmatrix} -8-(-4) & -1 \\ 16 & -(-4) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} -4 & -1 \\ 16 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\Rightarrow -4x_1 - x_2 = 0 \Rightarrow x_2 = -4x_1$

let $x_1 = 1, \Rightarrow x_2 = -4$

$\therefore$ $\boxed{\lambda_1 = -4, \quad \vec{v}_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}}$

53) $\begin{matrix} & + & - & + \end{matrix}$

$$\left| \begin{bmatrix} -\lambda & 4 & 0 \\ -1 & -4-\lambda & 0 \\ 0 & 0 & -2-\lambda \end{bmatrix} \right| = 0$$

$$\Rightarrow -\lambda \begin{vmatrix} -4-\lambda & 0 \\ 0 & -2-\lambda \end{vmatrix} - 4 \begin{vmatrix} -1 & 0 \\ 0 & -2-\lambda \end{vmatrix} + 0 \begin{vmatrix} -1 & -4-\lambda \\ 0 & 0 \end{vmatrix} = 0$$

$$\Rightarrow -\lambda((-4-\lambda)(-2-\lambda)-0) - 4((-1)(-2-\lambda)-0) + 0 = 0$$

$$\Rightarrow -\lambda[8+4\lambda+2\lambda+\lambda^2] - 8 - 4\lambda = 0$$

$$\Rightarrow -8\lambda - 6\lambda^2 - \lambda^3 - 8 - 4\lambda = 0$$

$$\Rightarrow \lambda^3 + 6\lambda^2 + 12\lambda + 8 = 0 \Rightarrow (x+2)^3 = 0$$

use Factor function on TI.

$$\Rightarrow \lambda = -2, -2, -2$$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 0 \\ -1 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \Rightarrow \begin{matrix} 2x_1 + 4x_2 = 0 \\ \Rightarrow x_1 = -2x_2 \\ \Rightarrow x_2 = 1,\ x_1 = -2 \\ x_3 = 0 \end{matrix}$$

$$\Rightarrow \boxed{\lambda = -2,\ v_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}} \text{ also } \boxed{v_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$$

works