Homework Solution

Section 4.5b
Undetermined Coefs.
(149) 1, 4, 12, 19, 28

1) ① $y_H'' + 3y_H' + 2y_H = 0 \Rightarrow (D^2 + 3D + 2)\, y_H = 0$

$\Rightarrow (D+2)(D+1) = 0 \Rightarrow D = -2, -1$

$\Rightarrow \boxed{y_H = c_1 e^{-2x} + c_2 e^{-x}}$

② $\boxed{y_c = A}$ ✓ ③

④ $y'' = 0, y' = 0 \Rightarrow y_c'' + 3y_c' + 2y_c = 2A = 6 \Rightarrow A = 3$

$\therefore \boxed{y_c = 3}$

⑤ $\boxed{y = c_1 e^{-2x} + c_2 e^{-x} + 3}$

12) ① $y_H'' - 16y_H = 0 \Rightarrow (D^2 - 16)\, y_H = 0$

$\Rightarrow (D+4)(D-4)\, y_H = 0 \Rightarrow D = \pm 4$

$\therefore \boxed{y_H = c_1 e^{4x} + c_2 e^{-4x}}$

② $\boxed{y_c = Ax e^{4x}}$ ③!! $\Rightarrow \boxed{y_c = Axe^{4x}}$

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4) $y_c' = Ae^{4x} + 4Axe^{4x}$, $y_c'' = 4Ae^{4x} + 4Ae^{4x} + 16Axe^{4x}$

$= 8Ae^{4x} + 16Axe^{4x}$

④ $y_c'' - 16y_c = 8Ae^{4x} + 16Axe^{4x} - 16Axe^{4x} = 2e^{4x}$

$\Rightarrow 8Ae^{4x} = 2e^{4x} \Rightarrow \boxed{A = \frac{1}{4}}$

⑤ $y = c_1e^{4x} + c_2e^{-4x} + \frac{1}{4}xe^{4x}$

4) ① $y_H'' + y' - 6y = 0 \Rightarrow (D^2 + D - 6)y = 0$

$\Rightarrow (D+3)(D-2)y_H \Rightarrow D = -3, 2$

$\therefore \boxed{y_H = c_1e^{-3x} + c_2e^{2x}}$

② $\boxed{y_c = Ax + B}$ ③ ✓✓

④ $y_c' = A \quad y_c'' = 0$

$\Rightarrow 0 + A - 6Ax - 6B = 2x$

$\Rightarrow (A - 6B) - 6Ax = 2x$

$\Rightarrow A = 1/3 \Rightarrow 1/3 - 6B = 0 \Rightarrow B = 1/18$

$\therefore y_c = \frac{1}{3}x + \frac{1}{18}$

⑤ $\boxed{y = c_1e^{-3x} + c_2e^{2x} + \frac{1}{3}x + \frac{1}{18}}$

19) ① $(D^2+2D+1)y_H = 0 \Rightarrow (D+1)^2 y_H = 0$

$\Rightarrow D = -1, -1 \Rightarrow \boxed{y_H = c_1 e^{-x} + c_2 x e^{-x}}$ ③ ✓✓

② $\boxed{y_C = A\sin(x) + B\cos(x) + C\sin(2x) + D\cos(2x)}$

④ $y_C' = A\cos(x) - B\sin(x) + 2C\cos(2x) - 2D\sin(2x)$

$y_C'' = -A\sin(x) - B\cos(x) - 4C\sin(2x) - 4D\cos(2x)$

$y_C'' + 2y_C' + y =$

$-A\sin(x) - B\cos(x) - 4C\sin(2x) - 4D\cos(2x)$
$+2A\cos(x) - 2B\sin(x) + 4C\cos(2x) - 4D\sin(2x)$
$+A\sin(x) + B\cos(x) + C\sin(2x) + D\cos(2x)$

$= (-A - 2B + A)\sin(x) + (-B + 2A + B)\cos(x)$

$(-4C - 4D + C)\sin(2x) + (-4D + 4C + D)\cos(2x)$

$= -2B\sin(x) + 2A\cos(x) + (-3C - 4D)\sin(2x) + (-3D + 4C)\cos(2x)$

$= \sin(x) + 3\cos(2x)$

$\Rightarrow -2B = 1 \Rightarrow B = -1/2$

$2A = 0 \Rightarrow A = 0$

$-3C - 4D = 0 \Rightarrow -12C - 16D = 0$

$4C - 3D = 3 \quad 12C - 9D = 9$

$-25D = 9 \quad D = -9/25$

$\Rightarrow -3C = -36/25 \Rightarrow C = +12/25$

$\therefore y_c = -\frac{1}{2}\cos(x) + \frac{12}{25}\sin(2x) + \frac{9}{25}\cos(2x)$

(5)

$\Rightarrow \boxed{y = c_1 e^{-x} + c_2 x e^{-x} - \frac{1}{2}\cos(x) + \frac{12}{25}\sin(2x) - \frac{9}{25}\cos(2x)}$

28) (1) $(2D^2 + 3D - 2)y_H = 0$

$(2D - 1)(D + 2) = 0 \Rightarrow D = \frac{1}{2}, -2$

$\boxed{y_H = c_1 e^{\frac{1}{2}x} + c_2 e^{-2x}}$ (3) ✓

(2) $\boxed{y_c = Ax^2 + Bx + C}$

(4) $y_c' = 2Ax + B$

$y_c'' = 2A$

$\Rightarrow 2y_c'' + 3y_c' - 2y_c = 14x^2 - 4x - 11$

$\Rightarrow 4A + 6Ax + 3B - 2Ax^2 - 2Bx - 2C = 14x^2 - 4x - 11$

$-2A = 14 \Rightarrow A = -7$

$6A - 2B = -4 \Rightarrow -42 - 2B = -4 \Rightarrow -2B = 38 \Rightarrow B = -19$

$4A + 3B - 2C = 11 \Rightarrow -28 - 57 - 2C = 11 \Rightarrow -2C = 74$

$\Rightarrow C = -37 \Rightarrow y_c = -7x^2 - 19x - 87$

20. $y = c_1 e^{\frac{1}{2}x} + c_2 e^{-2x} - 7x^2 - 19x - 37$

$y(0) = c_1 + c_2 - 37 = 0 \Rightarrow c_1 + c_2 = 37$

$y' = \frac{1}{2}c_1 e^{\frac{1}{2}x} - 2c_2 e^{-2x} - 14x - 19$

$y'(0) = \frac{1}{2}c_1 - 2c_2 - 19 = 0 \Rightarrow \frac{1}{2}c_1 - 2c_2 = 19$

$\Rightarrow c_1 - 4c_2 = 38$

$\Rightarrow \quad c_1 + c_2 = 37$

$-\ (c_1 - 4c_2 = 38)$

$5c_2 = -1 \Rightarrow \boxed{c_2 = -1/5}$

$c_1 = 37 + 1/5 \Rightarrow 186/5$

$$\boxed{y = \frac{186}{5}e^{\frac{1}{2}x} - \frac{1}{5}e^{-2x} - 7x^2 - 19x - 37}$$

CHECK

$y(0) = -1/5 + 186/5 - 37 = 0$ ✓✓

$y' = \frac{186}{10}e^{\frac{1}{2}x} + \frac{2}{5}e^{-2x} - 19$

$\Rightarrow y'(0) = \frac{186}{10} + \frac{2}{5} - 19 = 0$ ✓✓