Homework Solution

Section 5.1
(p14) 1, 2, 3, 5

1) $g = 32 ft/sec^2$

$m = F/g = 4/32 = 1/8$

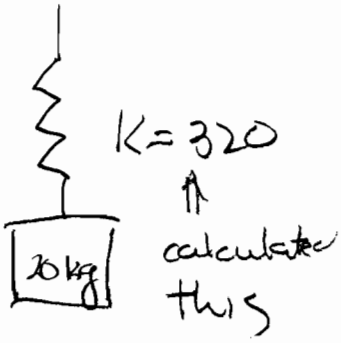
$m\ddot{x} + kx = 0 \Rightarrow \frac{1}{8}\ddot{x} + 16x = 0 \Rightarrow \ddot{x} + 128x = 0$

$\Rightarrow (D^2 + 128)x = 0 \Rightarrow D^2 + 128 = 0 \Rightarrow D^2 = -128 \Rightarrow D = \pm 8\sqrt{2}\,i$

$\Rightarrow x = C_1 \sin(8\sqrt{2}t) + C_2 \cos(8\sqrt{2}t)$

$p = 2\pi/\omega = 2\pi/8\sqrt{2} = \frac{2\sqrt{2}\pi}{16} = \boxed{\frac{\sqrt{2}\pi}{8}}$

2) a) $m\ddot{x} + kx = 0 \Rightarrow \ddot{x} + \frac{k}{20}x = 0$



$\Rightarrow D^2 + \frac{k}{20} = 0 \Rightarrow D^2 = -\frac{k}{20} \Rightarrow$

$D = \pm\sqrt{\frac{k}{20}}\,i$

ω

$f = \omega/2\pi = \sqrt{k/20}/2\pi = 2/\pi \Rightarrow \sqrt{k/20} = 4$

$\Rightarrow \frac{k}{20} = 16 \Rightarrow \boxed{k = 320}$

b) If $k = 80 \Rightarrow \omega = \sqrt{80/20} = \sqrt{4} = 2$

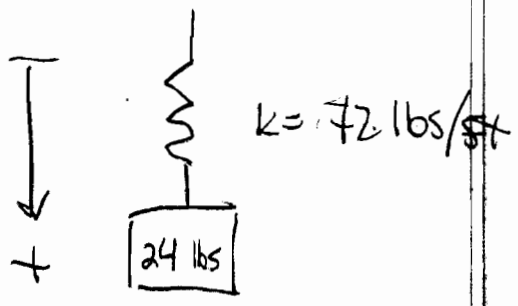
$\therefore f = \omega/2\pi = 2/2\pi = \boxed{1/\pi}$

3) $F = kx$

$\downarrow$

$24\text{lbs} = k(4\text{in}) \Rightarrow \boxed{k = 6\ \text{lbs/in}}$

$\Rightarrow k = 72\ \text{lbs/ft}$



If equilibrium is $x=0$ and down is positive

the $x(0) = -3$ inches $(3''$ up$) = -1/4$ ft

$x'(0) = 0$ (no initial velocity)

$\Rightarrow m\ddot{x} + kx = 0 \Rightarrow \frac{24}{32}\ddot{x} + 72x = 0$

$\Rightarrow \frac{3}{4}\ddot{x} + 72x = 0 \Rightarrow 3\ddot{x} + 288x = 0$

$\Rightarrow \ddot{x} + 96x = 0 \Rightarrow D^2 + 96 = 0 \Rightarrow D = \pm 4\sqrt{6}i$

$\Rightarrow x = C_1 \sin(4\sqrt{6}t) + C_2 \cos(4\sqrt{6}t)$

$x(0) = C_1(0) + C_2(1) = -1/4 \Rightarrow C_2 = -1/4$

$x = C_1 \sin(4\sqrt{6}t) - \frac{1}{4}\cos(4\sqrt{6}t)$

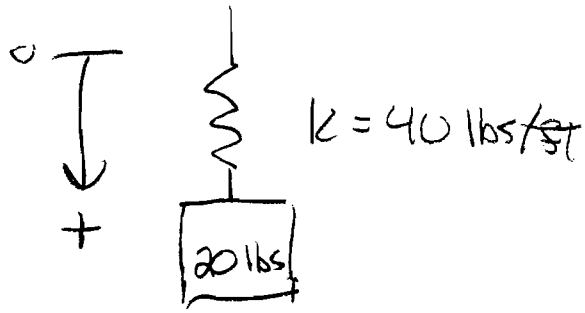
$x' = 4\sqrt{6}C_1 \cos(4\sqrt{6}t) + \sqrt{6}\sin(4\sqrt{6}t)$

$x'(0) = 4\sqrt{6}C_1(1) + \sqrt{6}(0) = 0 \Rightarrow \underline{C_1 = 0}$

$$\boxed{y = -\frac{1}{4}\cos(4\sqrt{6}t)}$$

5) $F = kx \Rightarrow 20 = k(\frac{1}{2}ft)$

$\Rightarrow k = 40$ lbs/ft



$x(0) = 1/2$ ft, $y'(0) = 0$

$m\ddot{x} + kx = 0 \Rightarrow \frac{20}{32}\ddot{x} + 40x = 0$

$\Rightarrow \ddot{x} + 64x = 0 \Rightarrow x = C_1 \sin(8t) + C_2 \cos(8t)$

$x(0) = C_2 = 1/2 \Rightarrow x = 8C_1 \cos(8t) - 4\sin(8t) = 0$

$\Rightarrow x'(0) = 8C_1(0) - 4(0) \Rightarrow C_1 = 0$

$\Rightarrow \boxed{x = \frac{1}{2}\cos(8t)}$

a) $x(\pi/12) = 1/2 \cos\left(\frac{2\pi}{3}\right) = 1/2(-1/2) = -1/4$

$x(\pi/8) = \frac{1}{2}\cos(\pi) = -1/2$

$x(\pi/6) = \frac{1}{2}\cos(4\pi/3) = -1/4$

$x(\pi/4) = \frac{1}{2}\cos(2\pi) = 1/2$

$x(9\pi/32) = \frac{1}{2}\cos(9\pi/4) = \frac{1}{2}\cos(\pi/4) = \sqrt{2}/4$

b) $x' = -4\sin(8t)$ (velocity)

$\Rightarrow x'(3\pi/16) = -4\sin(3\pi/2) = 4$ ft/sec

↑ downward

c) $\frac{1}{2}\cos(8t) = 0 \Rightarrow \cos(8t) = 0$

$\Rightarrow 8t = \frac{2n+1}{2}\pi = \boxed{t = \frac{2n+1}{16}\pi}$

↑ $\cos(t) = 0$ for $t = \pi/2, 3\pi/2, 5\pi/2, 7\pi/2, \ldots$