Section 5.1.3

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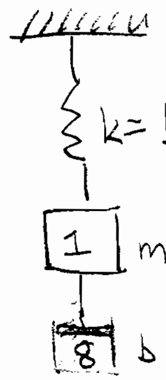
31)



$F = kx \Rightarrow \underset{mg}{(1\ \text{slug})(32)} = k(2\ \text{ft})$ ↙ given

$\Rightarrow k = 16$ lbs/ft

$\therefore \quad x'' + 8x' + 16x = 8\sin(4t)$

<u>Homogeneous</u> <u>Solution</u>

$(D^2 + 8D + 16)x = 0 \Rightarrow (D+4)^2 x = 0 \Rightarrow D = -4, -4$

$\therefore \quad x = c_1 e^{-4t} + c_2 t e^{-4t}$

<u>Particular</u> <u>Solution</u>

$(D^2 + 16)$ annihilates $8\sin(4t)$

$\therefore \quad x_p = A\sin(4t) + B\cos(4t)$

$x_p' = 4A\cos(4t) - 4B\sin(4t)$

$x_p'' = -16A\sin(4t) - 16B\cos(4t)$

$\Rightarrow (D^2 + 8D + 16)x_p = (\cancel{-16A} - 32B + \cancel{16A})\sin(4t)$
$+ (\cancel{-16B} + 32A + \cancel{16B})\cos(4t) = 8\sin(4t)$

$\Rightarrow B = -1/4, \quad A = 0 \Rightarrow x_p = -1/4\cos(4t)$

$\Rightarrow x = x_h + x_p$

$\Rightarrow \boxed{x = c_1 e^{-4t} + c_2 t e^{-4t} - \frac{1}{4}\cos(4t)}$

Apply Initial Values

these are implied to be

$$x(0) = 0, \quad x'(0) = 0$$

i.e. at equilibrium when force is applied

$$X(0) = c_1 \underbrace{e^0}_{\to 1} \; c_2 \underbrace{\cancel{(0)} e^0}_{\to 0} - \frac{1}{4} \underbrace{\cos(0)}_{\to 1} = 0$$

$$\Rightarrow c_1 = 1/4$$

$$\Rightarrow x = \frac{1}{4} e^{-4t} + c_2 t e^{-4t} - \frac{1}{4}\cos(4t)$$

$$\Rightarrow x' = -e^{-4t} + c_2 e^{-4t} - 4c_2 t e^{-4t} + \sin(4t)$$

$$\Rightarrow x'(0) = -1 + c_2 = 0 \Rightarrow c_2 = 1$$

$$\therefore \boxed{x = \frac{1}{4} e^{-4t} + t e^{-4t} - \frac{1}{4}\cos(4t)}$$

39) a) Solve homogeneous problem:

$$(D^2 + \omega^2) x = 0 \Rightarrow x = c_1 \cos(\omega t) + c_2 \sin(\omega t)$$

Solve particular solution

$(D^2 + \gamma^2)$ annihilates $F_0 \cos(\gamma t)$

$$\therefore \quad x_p = A\cos(\gamma t) + B\sin(\gamma t)$$

$$x_p' = -\gamma A \sin(\gamma t) + \gamma B \cos(\gamma t)$$

$$x_p'' = -\gamma^2 A\cos(\gamma t) - \gamma^2 B \sin(\gamma t)$$

$\therefore \quad (D^2 + \omega^2) x_p$

$$= (-\gamma^2 A + \omega^2 A)\cos(\gamma t) + (-\gamma^2 B + \omega^2 B)\sin(\gamma t)$$
$$= F_0 \cos(\gamma t)$$

$\Rightarrow (\omega^2 - \gamma^2) A = F_0 \qquad (\omega^2 - \gamma^2) B = 0 \Rightarrow B = 0$

$A = F_0 / (\omega^2 - \gamma^2)$

$\therefore \quad x_p = (F_0)/(\omega^2 - \gamma^2) \cos(\gamma t)$

$$\therefore \quad x = c_1 \cos(\omega t) + c_2 \sin(\omega t) + \frac{F_0}{(\omega^2 - \gamma^2)} \cos(\gamma t)$$

$$\therefore \quad x(0) = c_1(1) + c_2(0) + \frac{F_0}{(\omega^2 - \gamma^2)}(1) = 0$$

$$\Rightarrow c_1 = -\frac{F_0}{(\omega^2 - \gamma^2)}$$

$$\therefore \quad x = \frac{-F_0}{(\omega^2 - \gamma^2)} \cos(\omega t) + c_2 \sin(\omega t) + \frac{F_0}{(\omega^2 - \gamma^2)} \cos(\gamma t)$$

$$x' = \frac{+F_0 \omega}{(\omega^2 - \gamma^2)} \sin(\omega t) + \omega c_2 \cos(\omega t) - \frac{\gamma F_0}{(\omega^2 - \gamma^2)} \sin(\gamma t)$$

$$\Rightarrow x'(0) = 0 + \omega c_2 - 0 = 0 \Rightarrow c_2 = 0$$

$$\therefore \quad \boxed{x = \frac{F_0}{(\omega^2 - \gamma^2)} \left(\cos(\gamma t) - \cos(\omega t) \right)}$$

b) What happens when $\omega = \gamma$

$$\lim_{\gamma \to \omega} \frac{F_0}{\omega^2 - \gamma^2} \left(\cos(\gamma t) - \cos(\omega t) \right) \Rightarrow \infty (0)$$

Apply L'Hospitals rule

$$\lim_{\gamma \to \omega} \frac{\cos(\gamma t) - \cos(\omega t)}{(\omega^2 - \gamma^2)/F_0} = \frac{0}{0}$$

PROPER Form For L'Hospital's Rule

$$= \lim_{\gamma \to \omega} \frac{\frac{\partial}{\partial \gamma}\left(\cos(\gamma t) - \cos(\omega t) \right)}{\frac{\partial}{\partial \gamma}\left(\omega^2 - \gamma^2 / F_0 \right)} = \lim_{\gamma \to \omega} \frac{+t \sin(\gamma t)}{+2\gamma / F_0}$$

$$= \frac{t \sin(\omega t)}{2\omega / F_0} = \boxed{\frac{F_0}{2\omega} t \sin(\omega t) = x}$$

This is a physical problem because when $t \to \infty$, then $x \to \infty$. Basically the system blows up!!