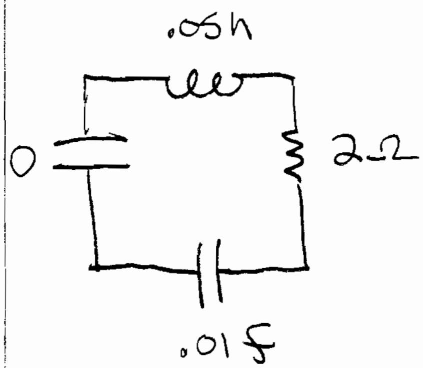
Homework Solutions

Section 5.1.4
(194) 45, 47, 53



$q(0)=5$
$i(0)=0 \Rightarrow q'(0)=0$

$$Lq'' + Rq' + \frac{1}{C}q = .05q'' + 2q' + 100q = 0$$

$$\Rightarrow q'' + 40q' + 2000q = 0 \Rightarrow (D^2 + 40D + 2000)q = 0$$

$$D = \frac{-40 \pm \sqrt{1600 - 8000}}{2} = \frac{-40 \pm \sqrt{-6400}}{2}$$

$$= \frac{-40 \pm 80i}{2} = -20 \pm 40i$$

$$\therefore q = C_1 e^{-20t}\cos(40t) + C_2 e^{-20t}\sin(40t)$$

$$q(0) = C_1(1)(1) + \cancel{C_2(1)(0)} = 5$$

$$\therefore q = 5e^{-20t}\cos(40t) + C_2 e^{-20t}\sin(40t)$$

$$q' = -100e^{-20t}\cos(40t) - 200e^{-20t}\sin(40t)$$
$$-20C_2 e^{-20t}\sin(40t) + 40C_2 e^{-20t}\cos(40t)$$

$$\Rightarrow q'(0) = -100 + 40C_2 = 0 \Rightarrow C_2 = \frac{100}{40} = 2.5$$

$$\therefore \boxed{q = 5e^{-20t}\cos(40t) + 2.5e^{-20t}\sin(40t)}$$

$$q = (A\sin(\omega t + \phi))\, e^{-20t}$$

$$A = (5^2 + 2.5^2)^{1/2} = 5.59$$

$$\phi = \tan^{-1}\left(\frac{C_{cos}}{C_{sin}}\right) = \tan^{-1}(2) = 1.11$$

$$\omega = 40$$

$$\therefore\ q = 5.59\, e^{-20t} \sin(40t + 1.11)$$

$$\boxed{q(.01) = 4.568 \text{ coulombes}}$$

b) $q = 0 \Rightarrow \sin(40t + 1.11) = 0$

$\Rightarrow 40t + 1.11 = n\pi$

$\Rightarrow$ for 1st 0, $\underline{n = 1}$ $\Rightarrow t = \dfrac{\pi - 1.11}{40}$

$\Rightarrow \boxed{t = .051 \text{ sec}}$

47) $Lq'' + Rq' + \frac{1}{C}q = E(t)$

IVs
$q(0) = 0$
$i(0) = 0 \Rightarrow q'(0) = 0$

$\Rightarrow \frac{5}{3}q'' + 10q' + 30q = 300$

$\Rightarrow 5q'' + 30q' + 90q = 900$

$\Rightarrow q'' + 6q' + 18q = 180$

Solve Homogeneous Solution

$(D^2 + 6D + 18)q = 0 \Rightarrow D = \frac{-6 \pm \sqrt{36-72}}{2}$

$\Rightarrow D = \frac{-6 \pm \sqrt{-36}}{2} = \frac{-6 \pm 6i}{3} = -3 \pm 3i$

$\therefore q_h = c_1 e^{-3t}\sin(3t) + c_2 e^{-3t}\cos(3t)$

Solve for particular solution

$\therefore q_p = A$
$q_p' = 0$
$q_p'' = 0$

$\Rightarrow (D^2 + 6D + 18)q_p = 180$

$\Rightarrow 18A = 180 \Rightarrow \boxed{A = 10}$

$\therefore q_p = 10$

$\Rightarrow q = c_1 e^{-3t}\sin(3t) + c_2 e^{-3t}\cos(3t) + 10$

$q(0) = c_2 + 10 = 0 \Rightarrow c_2 = -10$

$$\therefore q = c_1 e^{-3t}\sin(3t) - 10e^{-3t}\cos(3t) + 10$$

$$q' = -3c_1 e^{-3t}\sin(3t) + 3c_1 e^{-3t}\cos(3t)$$
$$+ 30e^{-3t}\cos(3t) + 30e^{-3t}\sin(3t)$$

$$\Rightarrow q'(0) = 3c_1 + 30 = 0 \Rightarrow c_1 = -10$$

$$\therefore q = -10e^{-3t}(\sin(3t) + \cos(3t)) + 10$$

$A = \sqrt{1^2+1^2} = \sqrt{2}$ $\quad \phi = \tan^{-1}(1) = \frac{\pi}{4}$

$$\therefore \boxed{q = -10\sqrt{2}\, e^{-3t}\sin(3t + \pi/4) + 10}$$

$$i = q' = 30\sqrt{2}\, e^{-3t}\sin(3t + \pi/4)$$
$$- 30\sqrt{2}\, e^{-3t}\cos(3t + \pi/4)$$

$$= 30\sqrt{2}\, e^{-3t}(\sin(3t + \pi/4) - \cos(3t + \pi/4))$$

$A = \sqrt{2}$ $\quad \phi = \tan^{-1}(-1) = -\pi/4$ $\quad \omega = 3$

$$\therefore \boxed{i = 60e^{-3t}(\sin 3t)}$$

max charge when $q'(t) = 0$ (or $i(t) = 0$)

$$\therefore \sin(3t) = 0 \Rightarrow 3t = 2\pi \Rightarrow t = 2\pi/3 \text{ sec}$$

$$\Rightarrow \boxed{q(2\pi/3) = 10.68\ c.}$$

chose 2π because this makes $\sin(3t + \pi/4) < 0$ thus making 1st term in q positive

53) $Lq'' + Rq' + \frac{1}{C}q = E(t)$

<u>IVs</u>

$q(0) = 1$

$i(0) = 0 \Rightarrow q'(0) = 0$

$\frac{1}{2}q'' + 10q' + 100q = 150$

$\Rightarrow q'' + 20q' + 200q = 300$

$\Rightarrow (D^2 + 20D + 200)q = 300$

<u>Solve Homogeneous PROBLEM</u>

$D = \frac{-20 \pm \sqrt{400-800}}{2} = \frac{-20 \pm \sqrt{-400}}{2} = \frac{-20 \pm 20i}{2} = -10 \pm 10i$

$\Rightarrow q_h = C_1 e^{-10t}\sin(10t) + C_2 e^{-10t}\cos(10t)$

<u>Solve for particular solution</u>

$\therefore q_p = A$

$\Rightarrow q_p' = 0$

$q_p'' = 0$

$(D^2 + 20D + 200)q = 300$

$\therefore 200A = 300 \Rightarrow A = 3/2$

$\therefore q = C_1 e^{-10t}\sin(10t) + C_2 e^{-10}\cos(10t) + 3/2$

$\Rightarrow q(0) = C_2 + 3/2 = 1 \Rightarrow C_2 = -\frac{1}{2}$

$$\Rightarrow q = c_1 e^{-10t}\sin(10t) - \frac{1}{2}e^{-10t}\cos(10t) + 3/2$$

$$\Rightarrow q' = -10c_1 e^{-10t}\sin(10t) + 10c_1 e^{-10t}\cos(10t) + 5e^{-10t}\cos(10t) + 5e^{-10t}\sin(10t)$$

$$\Rightarrow q'(0) = 10c_1 + 5 = 0 \Rightarrow c_1 = 1/2$$

$$\therefore q = -\frac{1}{2}e^{-10t}\left(\sin(10t) + \cos(10t)\right) + 3/2$$

$A = \sqrt{1^2 + 1^2} = \sqrt{2}$ $\quad \phi = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

$\omega = 10$

$$\therefore \boxed{q = -\frac{1}{2}\sqrt{2}e^{-10t}\sin(10t + \pi/4) + 3/2}$$

$\boxed{q(\infty) = 3/2}$ 1st term goes to zero