Homework Solutions

Section 8.1
(310) 1, 11, 13, 17, 21, 23

1) let $X = \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow X' = \begin{bmatrix} 3 & -5 \\ 4 & 8 \end{bmatrix} X$

11) $X = \begin{bmatrix} e^{-5t} \\ 2e^{-5t} \end{bmatrix} \quad X' = \begin{bmatrix} -5e^{-5t} \\ -10e^{-5t} \end{bmatrix}$

$$\begin{bmatrix} 3 & -4 \\ 4 & -7 \end{bmatrix} \begin{bmatrix} e^{-5t} \\ 2e^{-5t} \end{bmatrix} = \begin{bmatrix} -5e^{-5t} \\ -10e^{-5t} \end{bmatrix}$$

13) $X = \begin{bmatrix} -e^{-3t/2} \\ 2e^{-3t/2} \end{bmatrix} \Rightarrow X' = \begin{bmatrix} \frac{3}{2}e^{-3t/2} \\ -3e^{-3t/2} \end{bmatrix}$

$$X = \begin{bmatrix} -1 & 1/4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -e^{-3t/2} \\ 2e^{-3t/2} \end{bmatrix} = \begin{bmatrix} 3/2\, e^{-3t/2} \\ -3e^{-3t/2} \end{bmatrix}$$

17) Check to see is the vectors linearly independent by setting up Wronskian

$$\begin{vmatrix} e^{-2t} & e^{-6t} \\ e^{-2t} & -e^{-6t} \end{vmatrix} = W$$

$$\det(W) = (e^{-2t})(-e^{-6t}) - (e^{-6t})(e^{-2t})$$

$$= -e^{-8t} - e^{-8t} = -2e^{-8t} \neq 0$$

Since the vectors are linearly independent the form a fundamental set of solutions

21) $\Rightarrow \quad X_p = \begin{bmatrix} x_p \\ y_p \end{bmatrix} = \begin{bmatrix} 2t+5 \\ -t+1 \end{bmatrix}$

$\Rightarrow \quad X_p' = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

but $\quad X_p' = \begin{bmatrix} x_p + 4y_p + 2t - 7 \\ 3x_p + 2y_p - 4t - 18 \end{bmatrix}$

$$= \begin{bmatrix} 2t+5 - 4t + 4 + 2t - 7 \\ 6t + 15 - 2t + 2 - 4t - 18 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

23) $X_p' = \begin{bmatrix} e^t + te^t \\ e^t - te^t \end{bmatrix}$

$X_p' = \begin{bmatrix} 2e^t + te^t \\ -te^t \end{bmatrix}$

$X_p' = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} e^t + te^t \\ e^t - te^t \end{bmatrix} - \begin{bmatrix} e^t \\ 7e^t \end{bmatrix}$

$$= \begin{bmatrix} 2e^t + 2te^t + e^t - te^t - e^t \\ 3e^t + 3te^t + 4e^t - 4te^t - 7e^t \end{bmatrix} = \begin{bmatrix} 2e^t + te^t \\ -te^t \end{bmatrix}$$