Homework Solutions Section 9.4b
(357) #1, 2

1) $y'' - 4y' + 4y = 0, \quad y(0) = -2, \; y'(0) = 1$

<u>Analytic Solution:</u>

$$(D^2 - 4D + 4)y = 0 \Rightarrow (D-2)^2 y = 0$$

$$\therefore \; D = 2, 2 \Rightarrow \quad y = c_1 e^{2t} + c_2 t e^{2t}$$

$$\Rightarrow y(0) = c_1 = -2 \Rightarrow y = -2e^{2t} + c_2 t e^{2t}$$

$$y' = -4e^{2t} + c_2 e^{2t} + 2c_2 t e^{2t}$$

$$\Rightarrow y'(0) = -4 + c_2 = 1 \Rightarrow c_2 = 5$$

$$\therefore \; \boxed{y = -2e^{2t} + 5te^{2t}}$$

$$\Rightarrow y(.2) = -2e^{.4} + 5(.2)e^{.4}$$
$$= -e^{.4} \approx \boxed{-1.49}$$

<u>Numerical Solution:</u>

$u = y'$

$\Rightarrow u' - 4u + 4y = 0$

$$\Rightarrow \begin{cases} y' = u \\ u' = 4u - 4y \\ y(0) = -2 \\ u(0) = 1 \end{cases}$$

t	u	y	dy/dt	du/dt	Δt	Δu	Δy
0	1.0	-2.0	12	1.0	.1	1.2	.1
.1	2.2	-1.9	16.4	2.2	.1	1.64	.22
.2	3.84	-1.68					

$$\boxed{y(.2) \approx -1.68}$$

numerical approximation

2) $x^2y'' - 2xy' + 2y = 0$ $\Leftarrow$ <u>note</u>: this is a Cauchy Euler equation which you <u>do not</u> no how to solve analytically

$y(1) = 4,\ y'(1) = 9$

<u>Numerical Solution</u>

$$u' = y' \Rightarrow x^2u' - 2xu + 2y = 0$$
$$\Rightarrow u' = \frac{2}{x}u - \frac{2}{x^2}y.$$

i.e. System of equations

$y' = u$ $\qquad y(1) = 4$

$u' = (2/x)u - (2/x^2)y$ $\qquad u(1) = 9$

x	u	y	dy/dx	du/dx	Δx	Δu	Δy
1	9	4	10	9	.1	1.0	.9
1.1	10	4.9	10.08	10	.1	1.01	1.0
1.2	10.01	5.9					

$\therefore \boxed{y(1.2) \approx 5.9}$