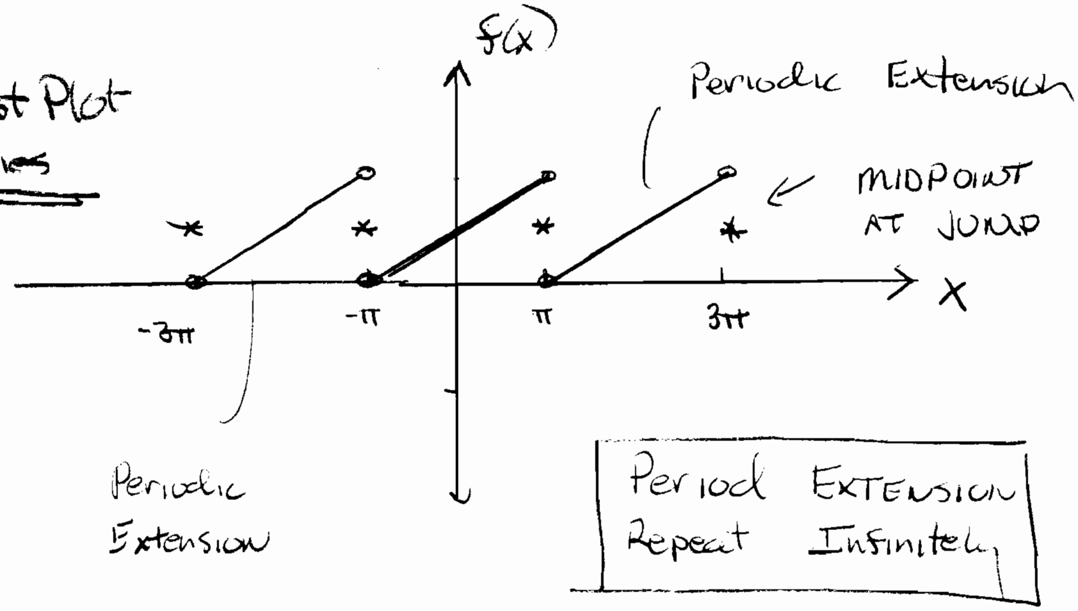
Homework Solutions

Section 11.2

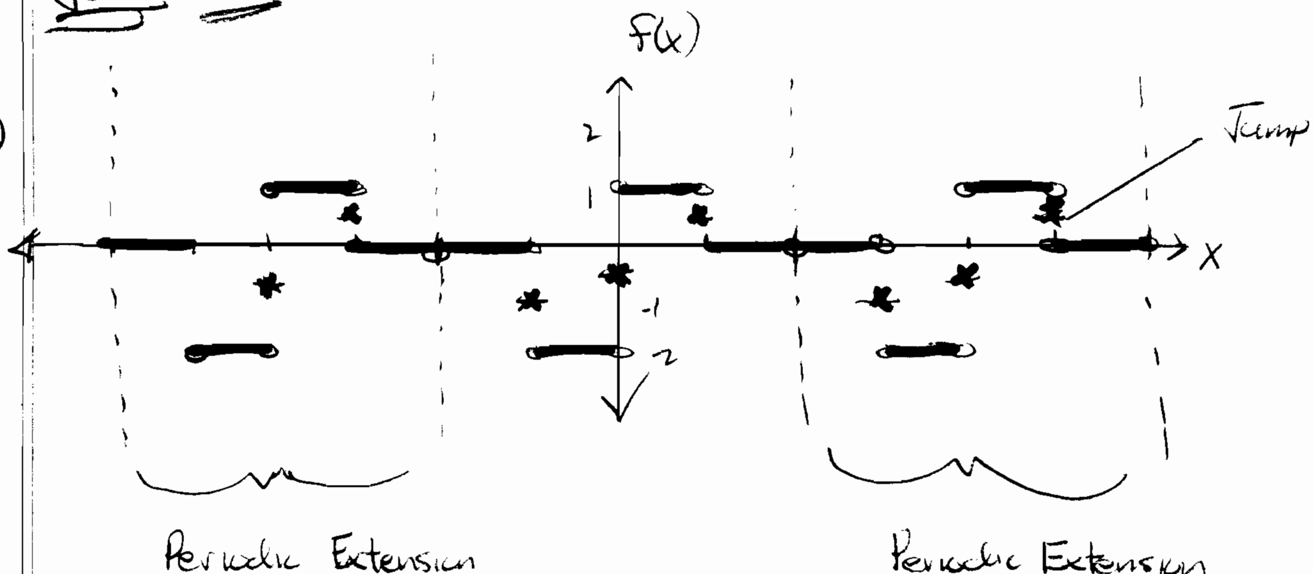
7, 11 (plot only), #17, 19

7) Just Plot Series

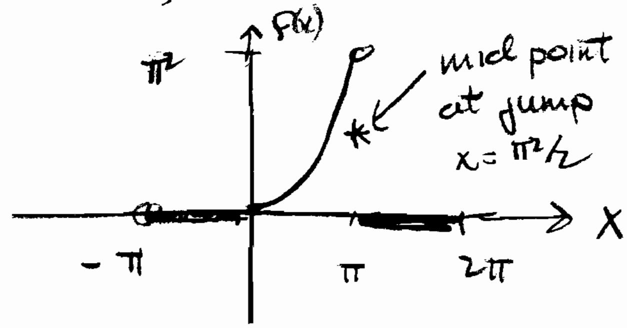


Just Plot

11)



17) (a) We know from (#5) the $f(x) = 0$ when $x = 0$



$\therefore \cos(nx) = \cos(0) = 1$
$\sin(nx) = \sin(0) = 0$

$$\therefore \quad 0 = \frac{\pi^2}{6} + \sum_{n=1}^{\infty} \frac{2(-1)^n}{n^2}$$

$$\Rightarrow \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = 0 \Rightarrow$$

$$\frac{\pi^2}{12} = -\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = -\left(-1 + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} \cdots\right)$$

$$\Rightarrow \boxed{\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \cdots\cdots}$$

(b) At $x = \pi$, there is a jump discontinuity
so $f(x) = (\pi^2 + 0)/2 = \pi^2/2$

$\therefore \cos(n\pi) = (-1)^n$
$\sin(n\pi) = 0$

$$\therefore \frac{\pi^2}{2} = \frac{\pi^2}{6} + \sum_{n=1}^{\infty} \frac{2(-1)^n}{n^2}(-1)^n \qquad d$$

$$\Rightarrow \frac{\pi^2}{4} = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n^2}$$ ← that just equals 1

$$\Rightarrow \boxed{\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}}$$

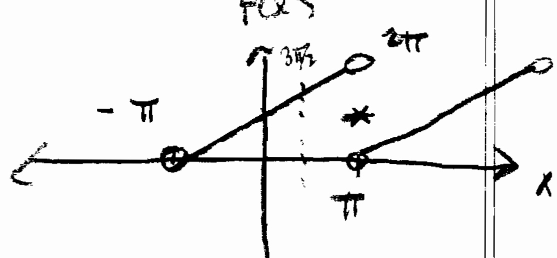
19) Since we dont have a solution for #7 from a previous homework, we will work it out

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+\pi)\,dx = 2\pi$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+\pi)\cos\left(\frac{n\pi x}{\pi}\right)dx = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+\pi)\sin(nx)\,dx = \overbrace{\cancel{\frac{2\sin(n\pi)}{n^2\pi}}}^{0} - \frac{2\overbrace{\cos(n\pi)}^{(-1)^n}}{n}$$

$$\therefore\ f(x) = \underbrace{\pi}_{a_0/2} - \sum_{n=1}^{\infty}\frac{2(-1)^n}{n}\sin(nx)$$



when $x = \pi/2$, $f(x) = 3\pi/2$

$$\therefore\ \frac{3\pi}{2} = \pi - \sum_{n=1}^{\infty}\frac{2(-1)^n}{n}\sin\left(\frac{n\pi}{2}\right)$$

$$\Rightarrow \frac{\pi}{2} = -2\left[\frac{(-1)}{1}\overbrace{\sin\left(\frac{\pi}{2}\right)}^{1} + \frac{(-1)^2}{2}\overbrace{\cancel{\sin(\pi)}}^{0} + \frac{(-1)^3}{3}\overbrace{\sin\left(\frac{3\pi}{2}\right)}^{-1} + \frac{(-1)^4}{4}\overbrace{\cancel{\sin(2\pi)}}^{0} + \frac{(-1)^5}{5}\underbrace{\sin\left(\frac{5\pi}{2}\right)}_{-1}\ \ldots\right)$$

$$\Rightarrow \boxed{\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \underbrace{\frac{1}{7} + \frac{1}{9} - \frac{1}{11}}_{}\ldots}$$

I'm guessing here because a pattern is emerging