Homework Solution

Section 11.3a

(414) 1, 3, 5, 7, 11, 13

(1) a) $f(x) = \sin(3x)$

b) $f(-x) = \sin(-3x) = -\sin(3x)$ [since $\sin(-\theta) = -\sin(\theta)$]

c) $\therefore f(-x) = -f(x) \Rightarrow$ **$f(x)$ is odd**

3) a) $f(x) = x^2 + x$

b) $f(-x) = (-x)^2 + (-x) = x^2 - x$

c) $f(x) \neq f(-x)$ and $f(x) \neq -f(-x)$

$\therefore$ **$f(x)$ is neither odd nor even**

5) a) $f(x) = e^{|x|}$

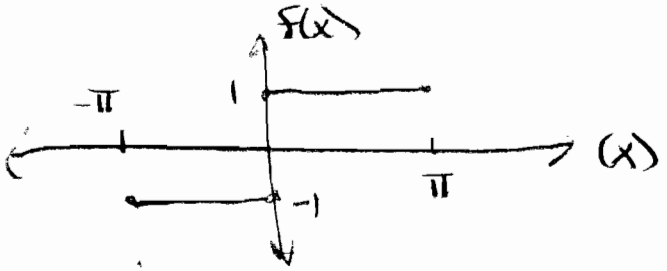
b) $f(-x) = e^{|-x|} = e^{|x|}$

c) $\therefore f(x) = f(-x) \Rightarrow$ **$f(x)$ is even**

7) a) $f(x) = x^2 \Rightarrow f(-x) = -x^2 = -f(x)$

b) $\therefore f(x) = -f(x)$ **$f(x)$ is odd**

11) $f(x)$ is a odd function



∴ use a sin expansion

$$b_n = \frac{2}{P}\int_0^P f(x)\sin\left(\frac{n\pi x}{P}\right)dx \quad \Rightarrow \boxed{P=\pi}$$

$$b_n = \frac{2}{\pi}\int_0^{\pi}(1)\sin(nx)dx = -\frac{2}{n\pi}\left(\cos(n\pi)-1\right)$$

$$\Rightarrow b_n = \frac{-2}{n\pi}\left[(-1)^n - 1\right] = \frac{2}{n\pi}\left[1-(-1)^{n+1}\right]$$

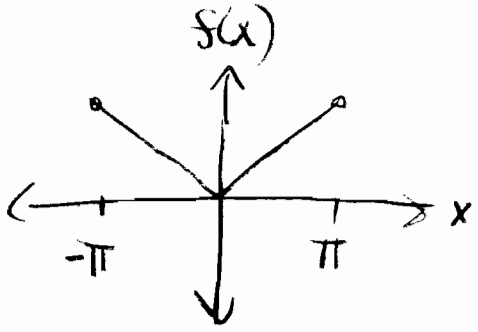
distribute -1 sign

$$\therefore \boxed{f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[1-(-1)^{n+1}\right]\sin(nx)}$$

$$\Rightarrow f(x) = \frac{2}{\pi}(0)\sin(x) + \frac{2}{2\pi}(2)\sin(2x) + \frac{2}{3\pi}(0)\sin(3x) + \frac{2}{4\pi}(2)\sin(4x) + \frac{2}{\pi(6)}(2)\sin(6x)$$

$$\boxed{f(x) \sim \frac{2}{\pi}\sin(2x) + \frac{1}{\pi}\sin(4x) + \frac{2}{3\pi}\sin(6x) + \frac{1}{2\pi}\sin(8x)\ldots}$$

13) $f(x) = |x|, \quad -\pi < x < \pi$



$$\Rightarrow f(x) = \begin{cases} -x & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

$\Rightarrow$ This is an even function $\Rightarrow$

i.e. $f(-x) = -(-x) = x = f(x)$

$\Rightarrow$ use cosine series

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x\,dx = \frac{2}{\pi}\cdot\frac{1}{2}(\pi^2 - 0) = \pi$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x\cos\left(\frac{n\pi x}{\pi}\right)dx$$

$$= \frac{2\cos(n\pi)}{n^2\pi} + \frac{2\sin(n\pi)}{n^2} - \frac{2}{n^2\pi}$$

($\cos(n\pi) = (-1)^n$; $\sin(n\pi) = 0$)

$$= \frac{2}{n^2\pi}\left((-1)^n - 1\right)$$

NOTE: this term is 0 for even numbers

$$\therefore \boxed{f(x) \sim \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n^2}\left((-1)^n - 1\right)\cos(nx)}$$

$a_0/2$

$$\therefore \boxed{f(x) \sim \frac{\pi}{2} - \frac{4}{\pi}\cos(x) - \frac{4}{9\pi}\cos(3x) - \frac{4}{25\pi}\cos(5x) \ldots}$$