Homework Solutions

Section 11.3
(414) 25, 29

25)

$$f(x) = \begin{cases} 1 & 0<x<1/2 \\ 0 & 1/2<x<1 \end{cases} \qquad p=1$$

Sine Series

$$b_n = \frac{2}{1}\int_0^{1/2} 1\sin(n\pi x)\,dx + \frac{2}{1}\int_{1/2}^{1} \cancel{0\sin(n\pi x)}^{\,0}\,dx$$

$$= \frac{2}{n\pi} - \frac{2}{n\pi}\cos\left(\frac{n\pi}{2}\right)$$ note: this = 0 when n is odd

$$\Rightarrow \boxed{f(x) \sim \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1}{n}\left(1 - \cos\left(\frac{n\pi}{2}\right)\right)\sin(n\pi x)}$$

Cosine Series

$$a_0 = \frac{2}{1}\int_0^{1/2} dx = \frac{2}{1}\left(\frac{1}{2}\right) = 1$$

$$a_n = \frac{2}{1}\int_0^{1/2} \cos(n\pi x)\,dx = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$\therefore \boxed{f(x) \sim \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\right)\cos(n\pi x)}$$

29)
$$f(x) = \begin{cases} x & 0 < x < \pi/2 \\ \pi - x & \frac{\pi}{2} \le x < \pi \end{cases} \qquad p = \pi \qquad \frac{n\pi x}{p} = nx$$

Sine Series

$$b_n = \frac{2}{\pi}\int_0^{\pi/2} (x)\sin(nx)\,dx + \frac{2}{\pi}\int_{\pi/2}^{\pi} (\pi - x)\sin(nx)\,dx$$

$$= \frac{2}{\pi}\left[\frac{2}{n^2}\sin\left(\frac{n\pi}{2}\right)\right] \Rightarrow b_n = \frac{4}{n^2\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$\Rightarrow \boxed{f(x) \sim \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{n^2}\sin\left(\frac{n\pi}{2}\right)\sin(nx)}$$

Cosine Series

$$a_0 = \frac{2}{\pi}\int_0^{\pi/2} x\,dx + \frac{2}{\pi}\int_{\pi/2}^{\pi} (\pi - x)\,dx = \frac{2}{\pi}\left(\frac{\pi^2}{4}\right) = \frac{\pi}{2}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi/2} x\cos(nx)\,dx + \int_{\pi/2}^{\pi} (\pi - x)\cos(nx)\,dx$$

$$= \frac{2}{\pi}\left[\frac{2}{n^2}\cos\left(\frac{n\pi}{2}\right) - (-1)^n\frac{1}{n^2} - \frac{1}{n^2}\right]$$

$$\boxed{f(x) = \frac{\pi}{4} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1}{n^2}\left[2\cos\left(\frac{n\pi}{2}\right) - (-1)^n - 1\right]\cos(nx)}$$

$a_0/2$