HOMEWORK SOLUTIONS

Section 6.1 - Power Series
(230) 1, 3, 8, 10, 11, 13

1) $$\lim_{n\to\infty}\left|\frac{C_{n+1}x^{n+1}}{C_n x^n}\right| = \lim_{n\to\infty}\left|\frac{C_{n+1}}{C_n}x\right| = |x|\lim_{n\to\infty}\left|\frac{C_{n+1}}{C_n}\right|$$

$$\Rightarrow \quad C_{n+1} = \frac{2^{n+1}}{n+1} \qquad C_n = \frac{2^n}{n}$$

$$\Rightarrow \quad \lim_{n\to\infty}\left|\frac{2^{n+1}}{n+1}\frac{n}{2^n}\right| = \lim_{n\to\infty}\left|2\frac{n}{n+1}\right| = 2\lim_{n\to\infty}\left|\frac{n}{n+1}\right| = 2$$

$$\Rightarrow \quad 2|x|<1 \Rightarrow |x|<\tfrac{1}{2} \Rightarrow \boxed{-\tfrac{1}{2}<x<\tfrac{1}{2}}$$

3) $$|x-5|\lim_{k\to\infty}\left|\frac{(-1)^{k+1}}{10^{k+1}}\frac{10^k}{(-1)^k}\right| = |x-5|\lim_{k\to\infty}\left|\frac{-1}{10}\right| < 1$$

$$\Rightarrow |x-5|\left(\frac{1}{10}\right)<1 \Rightarrow |x-5|<10$$

$$\Rightarrow \quad -10 < x-5 < 10 \Rightarrow \boxed{-5<x<15}$$

5)

$$\frac{1}{2}-\frac{3}{4}x+\frac{3}{8}x^2-\frac{3}{16}x^3 \cdots \Rightarrow \boxed{\frac{1}{2}+\sum_{n=1}^{\infty}(-1)^n\frac{3}{2^{n+1}}x^n}$$

$$2+x \,\big)\, 1-x$$

$$\underline{1+\tfrac{1}{2}x}$$

$$-\frac{3}{2}x$$

$$\underline{-\frac{3}{2}x-\frac{3}{4}x^2}$$

$$+\frac{3}{4}x^2$$

$$\underline{+\frac{3}{4}x^2+\frac{3}{8}x^3}$$

$\Rightarrow$ Radius of Convergence

$$|x| \lim_{n\to\infty}\left|\frac{(-1)^{n+1}(3)}{2^{n+2}} \cdot \frac{2^{n+1}}{(-1)^n(3)}\right| < 1$$

$$\Rightarrow |x| \lim_{n\to\infty}\left|-\frac{1}{2}\right| < 1 \Rightarrow \frac{1}{2}|x| < 1 \Rightarrow |x| < 2$$

$$\Rightarrow \boxed{-2 < x < 2}$$

10) $\sum_{n=3}^{\infty} (2n-1) c_n x^{n-3}$ let $k = n-3 \Rightarrow n = k+3$

$\Rightarrow n=3 \Rightarrow k=0$
$\Rightarrow n=\infty \Rightarrow k=\infty$

$$\sum_{k=0}^{\infty} (2(k+3)-1) c_{k+3} x^k = \boxed{\sum_{k=0}^{\infty} (2k+5) c_{k+3} x^k}$$

11) $$\sum_{n=1}^{\infty} 2n c_n x^{n-1} + \sum_{n=0}^{\infty} 6 c_n x^{n+1}$$

$k=n-1$
$n=k+1$
$n=1 \Rightarrow k=0$

$k=n+1$
$n=k-1$
$n=0 \Rightarrow k=1$

$$\sum_{k=0}^{\infty} 2(k+1) c_{k+1} x^k + \sum_{k=1}^{\infty} 6 c_{k-1} x^k$$

"peel off" $k=0$ term

$$2c_1 + \sum_{k=1}^{\infty} 2(k+1) c_{k+1} x^k + \sum_{k=1}^{\infty} 6 c_{k-1} x^k$$

combine

$$\boxed{2c_1 + \sum_{k=1}^{\infty} \left[(2k+2) c_{k+1} + 6 c_{k-1}\right] x^k}$$

13) $$y = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n$$

$$y' = \sum_{n=1}^{\infty} (-1)^{n+1} x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} (-1)^{n+1} (n-1) x^{n-2}$$

$$(x+1)y'' = \sum_{n=2}^{\infty} (-1)^{n+1} (n-1)(x^{n-1}) + \sum_{n=2}^{\infty} (-1)^{n+1} (n-1)(x^{n-2})$$

$$\Rightarrow \sum_{n=2}^{\infty} (-1)^{n+1} (n-1) x^{n-1} + \sum_{n=2}^{\infty} (-1)^{n+1} (n-1) x^{n-2} + \sum_{n=1}^{\infty} (-1)^{n+1} x^{n-1}$$

$k = n-1$, $n = 1+k$, $n=2 \Rightarrow k=1$

$k = n-2$, $n = 2+k$, $n=2 \Rightarrow 0$

$k = n-1$, $n = 1+k$, $n=1 \Rightarrow k=0$

$$\sum_{k=1}^{\infty} (-1)^k (k) x^k + \sum_{k=0}^{\infty} (-1)^{k+3} (k+1) x^k + \sum_{k=0}^{\infty} (-1)^{k+2} x^k$$

$$\sum_{k=1}^{\infty} (-1)^k (k) x^k + \cancel{-1(1) x^0} + \sum_{k=1}^{\infty} (-1)^{k+1} (k+1) x^k + \cancel{(1) x^0} \sum_{k=1}^{\infty} (-1)^k x^k$$

$$\sum_{k=1}^{\infty} \cancel{(-1)^k k x^k} + \cancel{(-1)^{k+1} k x^k} + \cancel{(-1)^{k+1} x^k} + \cancel{(-1)^k x^k} = 0$$

note: $(-1)^{k+3} = (-1)^{k+1}$ \$ $(-1)^{k+1} = -(-1)^k$

$(-1)^{k+2} = (-1)^k$