

Supplementary HW #3
Hermite Polynomials

1. $C_0 = (-2)^{4/2} (4-1)!! = (4)(3)(1) = 12$

$k=0$ $C_2 = \frac{2(0-4)}{(2)(1)} C_0 = -48$

$k=2$ $C_4 = \frac{2(2-4)}{(4)(3)} C_2 = \frac{(2)(-2)}{(4)(3)} (-48) = 16$

$$H_4(x) = 16x^4 - 48x^2 + 12$$

2. For $m=4 \Rightarrow$ Hermite D.E. is

$$y'' - 2xy' + 8y = 0$$

$$\Rightarrow y = 16x^4 - 48x^2 + 12$$

$$y' = 64x^3 - 96x$$

$$y'' = 192x^2 - 96$$

$$\begin{aligned} \Rightarrow 192x^2 - 96 + 128x^4 + 192x^2 + 128x^4 - 384x^2 + 96 \\ = 0 \end{aligned}$$

$$3) a) \int_{-\infty}^{\infty} (16x^4 - 48x^2 + 12)(16x^4 - 48x^2 + 12)e^{-x^2} dx = \boxed{384\sqrt{\pi}}$$

$$b) \int_{-\infty}^{\infty} (x^2 x_1) dx = 2^4 (4!) \sqrt{\pi} = \boxed{384\sqrt{\pi}}$$

$$4a) \int_{-\infty}^{\infty} (8x^3 - 12x)(16x^4 - 48x^2 + 12)e^{-x^2} dx = \boxed{0} \checkmark$$

$$b) \int_{-\infty}^{\infty} (x^2 x_1) dx = \boxed{0} \checkmark \text{ since } m \neq n$$