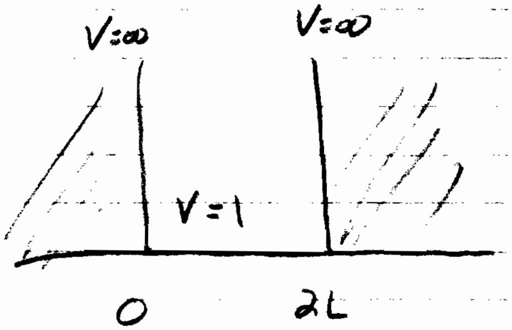
Supplementary Homework #1
Particle in a Box
Solution

1) $$\frac{2}{L}\int_{L/2}^{2L/3} \sin\left(\frac{2\pi x}{L}\right)^2 dx = .0977 \text{ or } 9.77\%$$

2)



$$-\frac{\hbar^2}{2m}\psi'' + \psi = E\psi$$

$$\Rightarrow \frac{\hbar^2}{2m}\psi'' + (E-1)\psi = 0$$

$$\Rightarrow \psi'' + \underbrace{\frac{2m(E-1)}{\hbar^2}}_{\uparrow\, k^2}\psi = 0$$

$$\Rightarrow \psi'' + k^2\psi = 0 \Rightarrow \psi = C_1\sin(kx) + C_2\cos(kx)$$

$$\therefore \psi(0) = C_1(0) + C_2(1) = 0 \Rightarrow \boxed{C_2 = 0}$$

$$\Rightarrow \psi = C_1\sin(kx) \Rightarrow \psi(2L) = C_1\sin(k(2L)) = 0$$

$$\Rightarrow 2kL = n\pi \Rightarrow \boxed{k = \frac{n\pi}{2L}}$$

$$\therefore \psi = C_1\sin\left(\frac{n\pi x}{2L}\right)$$

$\Rightarrow$ normalize $|\psi^2|$

$$\Rightarrow \int_0^{2L} C_1^2 \sin^2\left(\frac{n\pi x}{2L}\right) dx = 1$$

$$\Rightarrow C_1^2 L = 1 \Rightarrow C_1 = \frac{1}{\sqrt{L}}$$

$$\therefore \boxed{\psi = \frac{1}{\sqrt{L}} \sin\left(\frac{n\pi x}{2L}\right)}$$

$$\Rightarrow k^2 = \frac{2m(E-1)}{\hbar^2} = \frac{n^2\pi^2}{4L^2}$$

$$\Rightarrow E-1 = \frac{\hbar^2 n^2 \pi^2}{2m(4L^2)} \Rightarrow E = \frac{\hbar^2 n^2 \pi^2}{8mL^2} + 1$$

$$\hbar = \frac{h}{2\pi}$$

$$\Rightarrow E = \frac{h^2 n^2 \pi^2}{8mL^2(2^2\pi^2)} + 1 \Rightarrow \boxed{E = \frac{n^2 h^2}{32mL^2} + 1}$$

3) $$\int_{L/2}^{2L/3} \left(\frac{1}{\sqrt{L}} \sin\left(\frac{n\pi x}{2L}\right)\right)^2 dx$$

$$= \frac{1}{L} \int_{L/2}^{2L/3} \sin^2\left(\frac{n\pi x}{2L}\right) dx$$

NOTE $N=3$ FOR THE THIRD ENERGY LEVEL

$$\Rightarrow \frac{1}{L} \int_{L/2}^{2L/3} \sin^2\left(\frac{3\pi x}{2L}\right) dx \approx .0303$$

or $\boxed{3.03\%}$