Supplimentry HW #2
Gaussian Diff. Eq

1. $y = e^{-x^2/2}$

$y' = e^{-x^2/2} \frac{d}{dx}\left[\frac{-x^2}{2}\right] = -x e^{-x^2/2}$

$y'' = -e^{-x^2/2} + x^2 e^{-x^2/2}$

$\therefore y'' + (1-x^2)y = -e^{-x^2/2} + x^2 e^{-x^2/2} + (1-x^2)e^{-x^2/2}$

$= -e^{-x^2/2} + x^2 e^{-x^2/2} + e^{-x^2/2} - x^2 e^{-x^2/2}$

$= 0$ ✓✓

2. $y(0) = C_0(1) + C_1(0) = 0 \Rightarrow \boxed{C_0 = 0}$

$\therefore y = C_1\left(x - \frac{1}{6}x^3 + \frac{7}{120}x^5 - \frac{3}{560}x^7 \dots\right)$

$\Rightarrow y' = C_1\left(1 - \frac{1}{2}x^2 + \frac{7}{24}x^4 - \frac{3}{80}x^6\right)$

$\Rightarrow y'(0) = C_1(1) = 1 \Rightarrow C_1 = 1$

$\therefore y = x - \frac{1}{6}x^3 + \frac{7}{120}x^5 - \frac{3}{560}x^7 \dots$

$\Rightarrow \boxed{y = \frac{1}{2}\sqrt{\pi}\, e^{-x^2/2}\, erfi(x)}$

3) $k=6$: $C_8 = \frac{1}{(7)(6)}(C_4 - C_6)$

$= \frac{1}{56}\left(\frac{1}{8}C_0 + \frac{1}{48}C_0\right)$

$= \frac{1}{56}\left[\frac{6}{48} + \frac{1}{48}\right]C_0 = \frac{1}{56}\left[\frac{7}{48}\right]C_0$

$$\boxed{C_8 = \frac{1}{384}C_0}$$

$k=7$: $C_9 = \frac{1}{(8)(9)}(C_5 - C_7) = \frac{1}{72}\left(\frac{7}{120}C_1 + \frac{3}{560}C_1\right)$

$$C_9 = \frac{107}{120960}C_1$$

$\therefore$ $$\boxed{y = C_0\left(1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 - \frac{1}{48}x^6 + \frac{1}{384}x^8 \cdots\right] + C_1\left(x - \frac{1}{6}x^3 + \frac{7}{120}x^5 - \frac{3}{560}x^7 + \frac{107}{120,960}x^9 \cdots\right]}$$