Section 1.1 - INTRODUCTION
Lecture Notes

I **Definition:** Differential Equation

any equation contain derivatives:

i.e. $\frac{dy}{dx} = x \Rightarrow y = \frac{1}{2}x^2 + c$

$\frac{dy}{dx} + y = x \Rightarrow y = x - xe^{-x} + ce^{-x}$

$\frac{d^2y}{dx} + \frac{dy}{dx} + y = x \Rightarrow$

$c_1 e^{-\frac{1}{2}x} \sin\sqrt{3}x + c_2 e^{-\frac{1}{2}x} \cos\sqrt{3}x + x - 1$

Alternate Notations
$y' = x$
$y' + y = x$
$y'' + y' + y = x$

Note: we solve for dependent variable 'y' in terms of independent variable 'x'

II **Classification of DE**

A. **Type:** ORDINARY vs. Partial $\Rightarrow$

ORDINARY: 1 INDEPENDENT variable

Partial: >1 Independent variable

B. **ORDER:** i.e. Highest derivative

$\frac{d^2y}{dx^2} + y = x$ is 2^{nd} order

$(y'' + y = x)$

C. Linearity: linear vs non-linear

All terms involving dependent variable are to the 1^{st} power

Alt. Notation

$y'' + y^3 = x$: $\frac{d^2y}{dx^2} + y^3 = x$ non-linear

$y'' + y = x^2$ $\frac{d^2y}{dx^2} + y = x^2$ linear

$y'' + yy' = x$ $\frac{d^2y}{dx^2} + y\frac{dy}{dx} = x$ non-linear

III. Verification of Solution

A) Expicit: $y = 1/x$ is a solution to $xy' + y = 0$

note notation

$$\Rightarrow \left.\begin{aligned} y &= 1/x \\ y' &= \frac{dy}{dx} = -1/x^2 \end{aligned}\right\} \quad x\left(-\frac{1}{x^2}\right) + \frac{1}{x} = -\frac{1}{x} + \frac{1}{x} = 0 \checkmark$$

B) Implicit: $x^2 + y^2 = r^2$ solves $y\frac{dy}{dx} = -x$

note: non-linear

$$\Rightarrow \frac{d}{dx}\left[x^2 + y^2 = r^2\right] = 2x + 2y\frac{dy}{dx} = 0$$

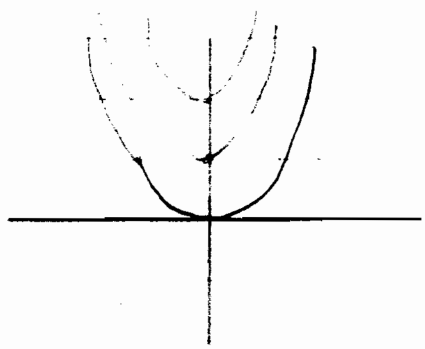
$$\Rightarrow 2y\frac{dy}{dx} = -2x \Rightarrow \boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

IV Families of Solutions

i.e. $\frac{dy}{dx} = 2x \Rightarrow y = x^2 + c$

$\Uparrow$

infinite # of parabolas depending on c



i.e. a differential equation has an infinite family of solutions

If I give you an "initial condition"

$\therefore \quad y(0) = 2 \Rightarrow y = 0^2 + c = 2 \Rightarrow c = 2$

$\Rightarrow \boxed{y = x^2 + 2} \leftarrow$ now I have one unique solution.