Lecture Notes

Section 2.1
Solution Curves

I) Recall General 1^{ST} ORDER IVP

$$\frac{dy}{dx} = f(x,y) \qquad y(x_0) = y_0$$

- The solution to the D.E. is a curve that goes through (x_0, y_0)
- Recall (from Calc I) that dy/dx is the slope of the curve

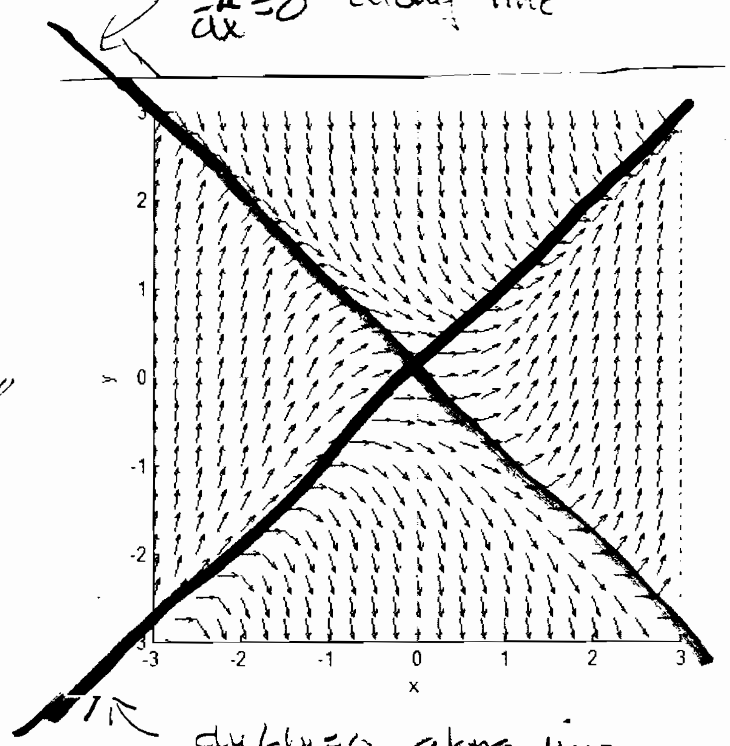
II) Direction Field:

- If I plot a vector representing a slope at various (x,y) coordinates, I produce a direction field.

i.e. consider

$$\frac{dy}{dx} = x^2 - y^2 \implies$$

$\frac{dy}{dx} = 0$ along line



$dy/dx = 0$ along line

i.e. at $(x,y) = (2,1)$

$\frac{dy}{dx} = 4 - 1 = 3$ "slope is 3"

note: slope = 0 when

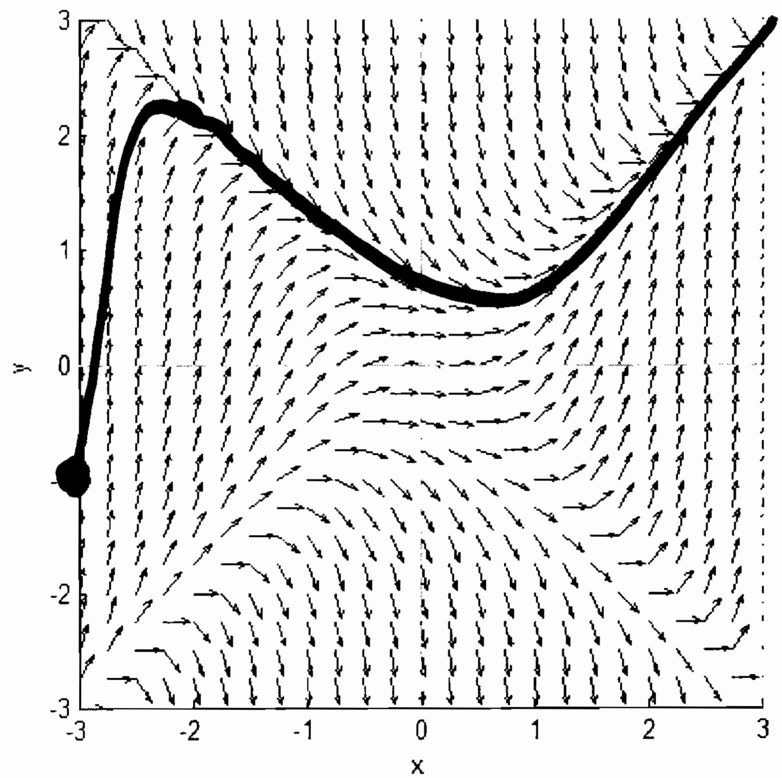
$x = y$ & $x = -y$

III. Solution Curve.

→ $\frac{dy}{dx} = x^2 - y^2$ by itself only produces direction field.

→ when I add initial condition (x_0, y_0), I get a solution curve.

i.e $\frac{dy}{dx} = x^2 - y^2$ subject to $y(-3) = -1$



IV Demonstration of Existence & Uniqueness

Consider:

$$\frac{dy}{dx} = x^2 - y^2$$ + several different intitial conditions

A) Existence: Each point produces a solution curve

B) Uniqueness: ① Each point that does not lie on a previous solution curve produces a unique solution curve

& ② Curves do not cross!