Lecture Notes

Section 2.6
Eulers Method

I) Recall 1ST ORDER IVP

Demo 603

$$\frac{dy}{dx} = x^2 - y^2 \qquad y(-3) = -2.8$$

Suppose I want to approximate $y(3)$
⇒ NOTE THAT I CAN'T SOLVE THE EQUATION ANALYTICALLY BECAUSE IT IS NON-LINEAR.

① FROM CALC I I know that $\frac{dy}{dx} \approx \frac{\Delta y}{\Delta x}$

$$\therefore \quad \Delta y \approx \frac{dy}{dx}(\Delta x)$$

I pick a Δx to get me from $x=-3$ to $x=3$

I have a D.E. THAT DEFINES dy/dx

② Say we pick $\Delta x = 1$ ⇒ Now set up a table to tabulate calculations

x	y	dx/dy *	Δx	= Δy ← ($\frac{dy}{dx}\Delta x$)
-3	-2.8	1.16 ↑ x^2-y^2	1 ↑ you chose this	1.61
Initial Condition				
-2	-1.19	2.58	1	2.58
$x+\Delta x$	$y+\Delta y$			
-1	1.39	-.93	1	-.93
0	.46	-.21	1	-.21
1	.25	.94	1	.94

→ Stop when you get to 3

II) This is better solved using EXCEL.

y'=x^2-y^2, y(-3)=-2.8, Δx=1						
x	y	y'	*	Δx	≈	Δy
-3.0000	-2.8000	1.1600	*	1.0000	≈	1.1600
-2.0000	-1.6400	1.3104	*	1.0000	≈	1.3104
-1.0000	-0.3296	0.8914	*	1.0000	≈	0.8914
0.0000	0.5618	-0.3156	*	1.0000	≈	-0.3156
1.0000	0.2462	0.9394	*	1.0000	≈	0.9394
2.0000	1.1856	2.5944	*	1.0000	≈	2.5944
3.0000	3.7800					

III) A Graphical Depiction of the Solution

Note: Eulers Method Only Approximates the solution

Note: A smaller Δx provides a better aproximation

Note: The computer used Δx=.001 to produce solution curve. (ie. 6000 vs 6 steps!)