

SM212 Lecture Notes

Section: 4.3 Homogeneous Linear Equations: General Soln.

Homework: (138) 3, 5, 15, 31, 35

Top Board

Eqn: $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = 0 \Leftarrow$ Homogeneous

Write, AS: $ay'' + by' + cy = 0$

Charact. Equation: $(aD^2 + bD + c)y = 0$ (Note: $D = \frac{d}{dx}$)

Has Roots $\left\{ \begin{array}{l} D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ D = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \end{array} \right.$

These are the key to constructing a solution

(I) Existence & Uniqueness

If $a, b, c, t_0, Y_0, Y_1 \in \mathbb{R}$
And: $ay'' + by' + cy = 0$ where $y(t_0) = Y_0$
 $y'(t_0) = Y_1$
Then there Exists a Unique Solution
for all $t \in (-\infty, \infty)$

II Discriminant: $b^2 - 4ac$

a) IF $b^2 - 4ac > 0 \Rightarrow r_1 \neq r_2 \in \mathbb{R}$ } Real
Non-Repeated
Roots
solution: $y(x) = c_1 e^{r_1 x} + c_2 e^{r_2 x}$

b) Example:

$$y'' + 2y' - 8y = 0 \Rightarrow y(0) = 3 \quad y'(0) = -12$$

$$(D^2 + 2D - 8)y = 0 \Rightarrow (D+4)(D-2) = 0 \Rightarrow D = -4, 2$$

$$y(x) = c_1 e^{-4x} + c_2 e^{2x} \Rightarrow y'(x) = -4c_1 e^{-4x} + 2c_2 e^{2x}$$

$$\Rightarrow y(0) = c_1 + c_2 = 3$$

$$\Rightarrow y'(0) = -4c_1 + 2c_2 = -12$$

$$6c_1 = 18 \Rightarrow \underline{\underline{c_1 = 3}} \quad c_2 = 0$$

$$\Rightarrow \underline{\underline{y(x) = e^{-4x}}}$$

c) IF $b^2 - 4ac = 0 \Rightarrow r_1 = r_2 \in \mathbb{R}$ } Real
Repeated
Roots
solution: $y(x) = c_1 e^{r_1 x} + c_2 x e^{r_1 x}$

d) Example

$$y'' + 4y' + 4 = 0 \Rightarrow y(0) = 0, \quad y'(0) = 1$$

$$(D^2 + 4D + 4)y = 0 \Rightarrow (D+2)^2 = 0 \Rightarrow D = -2, -2$$

$$\Rightarrow y(x) = c_1 e^{-2x} + c_2 x e^{-2x}$$

$$y(0) = c_1 = 0 \Rightarrow y(x) = c_2 x e^{-2x}$$

$$\Rightarrow y'(x) = c_2 e^{-2x} - 2c_2 x e^{-2x} \Rightarrow y'(0) = c_2 = 1$$

$$\Rightarrow y(x) = x e^{-2x}$$

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