

SM212 Lecture Notes

Section: 4.3b: 2<sup>nd</sup> Order Homo-DE's w/Complex Roots

Homework: (139) 9, 11, 25, 29, 43-48

Top Board

Recall :  $ay'' + by + c = 0$

Aux Eqn :  $(aD^2 + bD + c)y = 0$

Roots :  $r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Key to Constructing Solution

Ⓘ Solution: Discriminant  $b^2 - 4ac > 0$   
 $r_1 \neq r_2 \in \text{Real}$   
 $\Rightarrow y(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x}$

Ⓜ Solution: Discriminant:  $b^2 - 4ac = 0$   
 $r_1 = r_2 \in \text{Real}$   
 $\Rightarrow y(x) = C_1 e^{r_1 x} + C_2 x e^{r_1 x}$

III) what if  $b^2 - 4ac < 0$   $\in$  complex roots

$$\Rightarrow r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow r_1 \neq r_2 \in \text{Complex}$$

$$\Rightarrow \text{let } \alpha = \frac{-b}{2a} \quad \beta = \frac{\sqrt{4ac - b^2}}{2a} \leftarrow \text{why turned around?}$$

$$\therefore r_1 = \alpha + i\beta \quad r_2 = \alpha - i\beta$$

Solution

$$y(x) = C_1 e^{\alpha x} \sin \beta x + C_2 e^{\alpha x} \cos \beta x$$

IV) Example

$$y'' + 9 = 0 \Rightarrow (D^2 + 9)y = 0 \Rightarrow r = \frac{-0 \pm \sqrt{0 - 36}}{2}$$

$$\Rightarrow r = \frac{0}{2} \pm 3i = 0 \pm 3i \Rightarrow \alpha = 0, \beta = 3$$

$$y(x) = C_1 \sin 3x + C_2 \cos 3x$$

V) Example

$$w'' + 4w' + 6w = 0 \Rightarrow (D^2 + 4D + 6)w = 0$$

$$r = \frac{-4 \pm \sqrt{16 - 24}}{2} = \frac{-4 \pm \sqrt{-8}}{2} = \frac{-4 \pm 2\sqrt{-2}}{2}$$

$$= -2 \pm \sqrt{-2} = -2 \pm 2i \Rightarrow \alpha = -2 \quad \beta = 2$$

$$\Rightarrow y(x) = C_1 e^{-2x} \sin 2x + C_2 e^{-2x} \cos 2x$$