Lecture Notes

Section 5.1a
Mass/Spring Systems
Undamped Motion

I Hooke's Law $\Rightarrow F = kx$

F = force exerted by weight
($F = Mg$)
k = spring constant
x = amt. spring stretches (displacement)

Example: 10 lb weight stretches spring 6 inches
↑ this is a force in imperial sys

$10 = k(6) \Rightarrow k = \frac{5}{3}$ lbs/in $= 20$ lbs/ft

Example: 10 kg weight stretches spring 6 cm
↑ this is a mass in metric sys

$F = mg = (9.8)(10) = 98$ N.

$98 = k(6) \Rightarrow k = \frac{49}{3}$ N/cm $= \frac{4900}{3}$ N/m

II Free Undamped Motion: $\boxed{m\frac{d^2x}{dt^2} + kx = 0}$

$\Rightarrow x'' + \frac{k}{m}x = 0 \Rightarrow x'' + \omega^2 x = 0$
where $\omega^2 = k/m$

Solution:

$$\boxed{\begin{aligned} x &= c_1\cos(\omega t) + c_2\sin(\omega t) \\ &= c_1\cos\left(\sqrt{\tfrac{k}{m}}\,t\right) + c_2\sin\left(\sqrt{\tfrac{k}{m}}\,t\right) \end{aligned}}$$

III. Definitions

A. Frequency: cycles per second

$$f = \omega/2\pi = \sqrt{k/m}/2\pi$$

B. Period: seconds per cycle. = $1/f$

C. Circular or Natural Frequency = $\omega = \sqrt{k/m}$

IV Example: See Example 1 (183)

V Alternate form of solution

$$y = C_1 \sin(\omega t) + C_2 \cos(\omega t) = A \sin(\omega t + \phi)$$

ϕ: phase shift

$$A = (C_1^2 + C_2^2)^{1/2}$$

← constant associated w/ cos

$$\phi = \tan^{-1}\left(\frac{C_{\text{"cos"}}}{C_{\text{"sin"}}}\right)$$

← constant associate w/ sin

Note: If $C_{\sin} < 0$, add 180 (or π) to ϕ

i.e. $x = -\sqrt{3}\sin t + \cos(t) = \boxed{2\sin(t + 5\pi/6)}$

$A = \sqrt{3+1} = 2$

$\phi = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = -\frac{\pi}{6} + \pi = 5\pi/6$ ← since $C_{\sin} < 0$