Lecture Notes

Appendix II
Matrices: Definition & Theory

I. Definition: Matrix - Any ARRAY of Numbers, i.e.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

a_{ij}

row i, column j

a) Matrix A is 3×2
i.e. 3 rows by 2 columns

b) Scalar Multiplication

$$kA = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \\ ka_{31} & ka_{33} \end{bmatrix} \quad \text{i.e.} \quad 2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

c) TRANSPOSE OF Matrix

$$A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \end{bmatrix} \quad \text{i.e.} \quad \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

i.e. rows become columns

d) Definition: Column Matrix

$$X = \begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \\ \vdots \\ x_n \end{bmatrix}$$

note: X is $n \times 1$, i.e. 'n' rows and 1 column

II) Opperations between 2 MATRICES, Let

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

a) Addition/Subtraction

$$A \pm B = \begin{bmatrix} a_{11} \pm b_{11} & a_{12} \pm b_{12} \\ a_{21} \pm b_{21} & a_{22} \pm b_{22} \end{bmatrix}$$

<u>note</u>: matrices must be same size to add or subtract.

b) Multiplication

$$A * B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{21}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

i.e. $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} (1)(2) + 2(0) & (1)(-3) + (2)(1) \\ (-1)(2) + (0)(0) & (-1)(-3) + (0)(1) \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$$

$$BA = \begin{bmatrix} (2)(1) + (-3)(-1) & (2)(2) + (-3)(0) \\ (0)(1) + (1)(-1) & (0)(2) + (1)(0) \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ -1 & 0 \end{bmatrix}$$

<u>NOTE</u>: $AB \neq BA$

NOTE: WHEN CAN 2 MATRICES BE MULTIPLIED TOGETHER?

if A is $i \times j$ & B is $m \times n$

then $\underset{(i \times j)}{A} \times \underset{(m \times n)}{B}$ is possible iff $j = m$
result AB is $i \times n$

i.e. if A is 4×2 and B is 2×6

then AB is possible and will be 4×6
BA is not possible

i.e. $\underset{(2 \times 6)}{B} \times \underset{(4 \times 2)}{A}$

$6 \neq 4$ $\therefore$ cant multiply

III) Determinants of Matrices (Examples)

A) 2x2 MATRIX

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow Det(A) = (1)(4) - (2)(3)$$

product of diagonal

minus product of off-diagonal

B) <u>3x3 Matrix</u>

$$\begin{array}{ccc} + & - & + \\ \end{array}$$
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = 1\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$$

(RECALL TAKING CROSS PRODUCTS IN CALCULUS THREE).

$$= (1)(45-48) - (2)(36-42) + (3)(32-35)$$
$$= (1)(-3) - (2)(-6) + (3)(-3)$$
$$-3 + 12 - 9 = 0$$

IV) <u>A MATRIX OF Functions</u>

$$A(x) = \begin{bmatrix} f(x) & g(x) \\ h(x) & i(x) \end{bmatrix}$$

A. <u>Derivative of A(x)</u>

$$A'(x) = \begin{bmatrix} f'(x) & g'(x) \\ h'(x) & i'(x) \end{bmatrix}$$

i.e. $A = \begin{bmatrix} e^x \\ e^{3x} \\ e^{-2x} \end{bmatrix} \Rightarrow A' = \begin{bmatrix} e^x \\ 3e^{3x} \\ -2e^{-2x} \end{bmatrix}$