I. Strategies for Solving a System of Equations

$$3x + 2y = 7$$
$$x - 3y = -5$$

A. Substitution

① From 2nd Equation $x - 3y = -5 \Rightarrow x = 3y - 5$

② "Plug" $x = 3y - 5$ into 1st equation

$$\Rightarrow 3(3y-5) + 2y = 7 \Rightarrow 9y - 15 + 2y = 7$$
$$\Rightarrow 11y = 22 \Rightarrow \boxed{y = 2}$$

③ Now find $x \Rightarrow$ $x = 3y - 5 = 3(2) - 5 = 1 \Rightarrow \boxed{x = 1}$

B) Gaussian Elimination

$$3x + 2y = 7$$
$$x - 3y = -5$$

① Make a variable "go away", ie.
multiply equation 2 by 3 and subtract
equation 2 from equation 1

$$\left. \begin{array}{l} 3x + 2y = 7 \\ - (3x - 9x = -15) \end{array} \right\} \Rightarrow 11y = 22 \Rightarrow \boxed{y = 2}, \Rightarrow \text{find } 'x'$$

c. Using Augmented Matrix

① Rewrite System as Augmented Matrix

$$\begin{bmatrix} 3 & 2 & \vdots & 7 \\ 1 & -3 & \vdots & -5 \end{bmatrix}$$

coefficients of x and y   right hand side of equations

② We want to do row operations that will end in this matrix

$$\begin{bmatrix} 1 & 0 & \vdots & a \\ 0 & 1 & \vdots & b \end{bmatrix}$$

this will be the answer

③ Legal Row Operations are:

    ⓘ multiply row by non-zero scalar
    ⓤ linearly combine rows
    ⓥ swap rows.

④ $\begin{bmatrix} ③ & 2 & | & 7 \\ 1 & -3 & | & -5 \end{bmatrix}$ R1/3 (i.e. make circled element a 1)

$\Rightarrow \begin{bmatrix} 1 & 2/3 & | & 7/3 \\ ① & -3 & | & -5 \end{bmatrix}$ R2-R1 (i.e make circled element a zero)

$\Rightarrow \begin{bmatrix} 1 & 2/3 & 7/3 \\ 0 & -11/3 & -22/3 \end{bmatrix}$ R2 × $\frac{-11}{3}$ (i.e. make circled element a '1')

$\Rightarrow \begin{bmatrix} 1 & 2/3 & | & 7/3 \\ 0 & 1 & | & 2 \end{bmatrix}$ R1-2/3R₂ (i.e make circled element a '0')

$\begin{bmatrix} 1 & 0 & | & 1 \\ 0 & 1 & | & 2 \end{bmatrix}$

⇑ answer!! $x=1, y=2$

II Example 12. (APP 11)

STEPS ↓

$2x + 6y + z = 7$
$x + 2y - z = -1$
$5x + 7y - 4z = 9$

$\Rightarrow \begin{bmatrix} 2 & 6 & 1 & | & 7 \\ 1 & 2 & 1 & | & -1 \\ 5 & 7 & -4 & | & 9 \end{bmatrix}$

$\square \Rightarrow$ make these elements '1'

$\bigcirc \Rightarrow$ make these elements '0'

III Finally by Calculator (RREF Function)

$$\text{RREF}\left(\left[\,2,6,1,7\,;\ 1,2,-1,-1\,;\ 5,7,-4,9\,\right]\right)$$

brackets
indicate
matrix

semi-colon
separates
rows of
matrix