Appendix II
Inverse of a Matrix

I Definition

① A square matrix has an inverse iff $\det(A) \neq 0$.

② A^{-1} is the inverse of A

③ $\boxed{AA^{-1} = I_n}$ where I_n is the identity matrix

Note: It is also true that $A^{-1}A = I$

④ Indentity Matrix: A matrix whose diagonal values are 1 and all other elements are equal to 0.

i.e $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Example:

If $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

$$AA^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

II) FORMULA FOR FINDING INVERSE OF A 2×2

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

where $\det(A) \neq 0$.

i.e. $A = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow \mathrm{Det}(A) = 2$

$\Rightarrow A^{-1} = \frac{1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3/2 & 1 \end{bmatrix}$

III) MORE GENERAL METHOD (WORK FOR 2×2 & higher)

ⓐ SET UP Following MATRIX

IF $A = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow$ SET UP $\left[\begin{array}{cc|cc} 2 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right]$

Matrix A

Identity Matrix

ⓑ USE Row Operations to Turn Left SIDE OF MATRIX INTO THE IDENTITY MATRIX i.e.

$$\left[\begin{array}{cc:cc} 2 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right] \quad R1/2$$

$$\Rightarrow \left[\begin{array}{cc:cc} 1 & 1 & 1/2 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right] \quad R2-3R1$$

$$\Rightarrow \left[\begin{array}{cc:cc} 1 & 1 & 1/2 & 0 \\ 0 & 1 & -3/2 & 1 \end{array}\right] \quad R1-R2$$

already '1'!

$$\Rightarrow \left[\begin{array}{cc:cc} 1 & 0 & 2 & -1 \\ 0 & 1 & -3/2 & 1 \end{array}\right]$$

This is A^{-1}

IV Finally - By Calculator

inv([2,2;3,4])