

Example		
$\frac{dy}{dx} + y = e^x, y(0) = 1$		
Step 1	Solve for y_h (the homogeneous solution)	$\frac{dy_h}{dx} + y_h = 0 \rightarrow (D + 1)y_h = 0 \rightarrow$ $D = -1 \rightarrow y_h = c_1 e^{-x}$
Step 2	Guess a solution to y_p (based on the right-hand side of the original D.E.)	$y_p = Ae^x$
Step 3	Compare y_h to y_p . If any terms in y_p are the same as the terms in y_h , then multiply the “offending” terms in y_p by the independent variable	The terms in y_h and y_p are different, so do nothing.
Step 4	Solve for A, B, C, ... by substituting y_p into the original D.E.	$\frac{dy_p}{dx} + y_p = Ae^x + Ae^x = e^x \rightarrow$ $2Ae^x = e^x \rightarrow 2A = 1 \rightarrow A = \frac{1}{2} \rightarrow$ $y_p = \frac{1}{2}e^x$
Step 5	Combine y_h and y_p , i.e. $y = y_h + y_p$.	$y = c_1 e^{-x} + \frac{1}{2}e^x$
Step 6	Solve for $c_1, c_2, c_3 \dots$ by applying initial values	$y(0) = c_1 + \frac{1}{2} = 1 \rightarrow c_1 = \frac{1}{2} \rightarrow$ $y = \frac{1}{2}e^{-x} + \frac{1}{2}e^x$

Example		
$y'' + y = \sin(t), y(0) = 1, y'(0) = 0$		
Step 1	Solve for y_h (the homogeneous solution)	$y_h'' + y_h = 0 \rightarrow (D^2 + 1)y_h = 0 \rightarrow$ $D = \pm i \rightarrow y_h = c_1 \sin(t) + c_2 \cos(t)$
Step 2	Guess a solution to y_p (based on the right-hand side of the original D.E.)	$y_p = A \sin(t) + B \cos(t)$
Step 3	Compare y_h to y_p . If and terms in y_p are the same as the terms in y_h , then multiply the "offending" terms in y_p by the independent variable	The terms in y_h and y_p are the same, therefore: $y_p = At \sin(t) + B t \cos(t)$.
Step 4	Solve for A, B, C, ... by substituting y_p into the original D.E.	$y_p = At \sin(t) + B t \cos(t)$ $y_p' = A \sin(t) + A t \cos(t) + B \cos(t) - B t \sin(t)$ $y_p'' = 2A \cos(t) - A t \sin(t) - 2B \sin(t) - B t \cos(t)$ $y_p'' + y_p = 2A \cos(t) - 2B \sin(t) = \sin(t)$ $2A = 0$ and $-2B = 1 \rightarrow A = 0, B = -1/2$ $y_p = -\frac{1}{2} t \cos(t)$
Step 5	Combine y_h and y_p , i.e. $y = y_h + y_p$.	$y = c_1 \sin(t) + c_2 \cos(t) - \frac{1}{2} t \cos(t)$
Step 6	Solve for $c_1, c_2, c_3 \dots$ by applying initial values	$y(0) = c_2 = 1 \rightarrow$ $y = c_1 \sin(t) + \cos(t) - \frac{1}{2} t \cos(t) \rightarrow$ $y' = c_1 \cos(t) - \sin(t) - \frac{1}{2} \cos(t) - \frac{1}{2} t \sin(t) \rightarrow$ $y'(0) = c_1 - \frac{1}{2} = 0 \rightarrow c_1 = \frac{1}{2} \rightarrow$ $y = \frac{1}{2} \sin(t) + \cos(t) - \frac{1}{2} t \cos(t)$