

I) We have explored the homogeneous D.E

$$ay'' + by' + cy = 0$$

⇒ We now consider the non-homogeneous form of the equation:

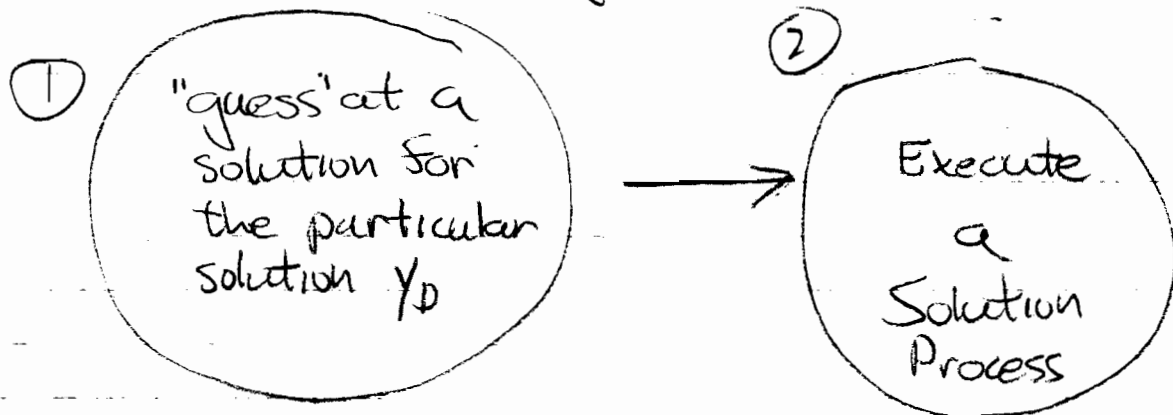
$$ay'' + by' + cy = g(x)$$

We can solve this equation if

$$g(x) = \begin{cases} \textcircled{1} \text{ polynomial} \\ \textcircled{2} \text{ exponential} \\ \textcircled{3} \text{ sin/cos} \end{cases}$$

or any additive or multiplicative combination of the function.

II) The method of undetermined coefficients consists of 2 major steps:



III) Today we focus on the Guess...

A) Primary Rules:

$g(x)$ is a polynomial, exponential, or sin/cos

1) i) $g(x) = 3x^2 - 2 \xrightarrow{\text{guess}} y_c = Ax^2 + Bx + C$

② 1st power term missing in $g(x)$ will not be missing in guess

① Start w/highest power in $g(x)$ and form full polynomial

③ Values of coefficients do not affect guess

④ USE A, B, C, ... for unknown coefficients

⑤ Stop at other power

ii) $g(x) = -100x^4 + 2x \xrightarrow{\text{guess}} y_c = Ax^4 + Bx^3 + Cx^2 + Dx + E$

2) Exponentials

i) $g(x) = e^x \xrightarrow{\text{guess}} y_c = Ae^x$

ii) $g(x) = -10e^{3x} + 50e^{-6x} \xrightarrow{\text{guess}} y_c = Ae^{3x} + Be^{-6x}$

① Include all different exponential terms in guess

3) Sin/Cos

i) $g(x) = 10 \sin(x) \xrightarrow{\text{guess}} y_c(x) = A \sin(x) + B \cos(x)$

① Must guess both sin & cos.

Polynomials

$$\text{ii) } q(x) = 2\sin(x) - 4\cos(x) \xrightarrow{\text{guess}} y_c(x) = A\sin(x) + B\cos(x)$$

① single guess covers both terms in $q(x)$

$$\text{iii) } q(x) = \sin(x) + \cos(2x) \xrightarrow{\text{guess}} y_c(x) = A\sin(x) + B\cos(x) + C\sin(2x) + D\cos(2x)$$

① THESE ARE DIFFERENT

B) Secondary Rules - Multiplicative combinations of two categories

1) polynomial * exponential

$$\text{(i) } q(x) = x^2 e^{2x} \xrightarrow{\text{guess}} y_c = Ax^2 e^x + Bx e^x + Cx$$

① start w/ highest power in $q(x)$ and continue to 0th power

$$\text{(ii) } q(x) = (x^3 + 1)e^x \xrightarrow{\text{guess}} Ax^3 e^x + Bx^2 e^x + Cx e^x + D e^x$$

2) polynomial * (sin/cos)

$$\text{(i) } q(x) = x^2 \sin(x) \xrightarrow{\text{guess}} Ax^2 \sin(x) + Bx^2 \cos(x) + Cx \sin(x) + Dx \cos(x) + E \sin(x) + F \cos(x)$$

① must "hide" highest power of polynomial to 0

② must include sin & cos in guess

3) exponential * (sin/cos)

$$i) -10e^x \sin(2x) \xrightarrow{\text{guess}} Ae^x \sin(ax) + Be^x \cos(ax)$$

① must guess sines & cosines

c) Tertiary Rules (multiplicative combination of all 3 elements)

$$i) g(x) = 10x^2 e^{ax} \sin(Bx) \xrightarrow{\text{guess}}$$

$$Ax^2 e^{ax} \sin(Bx) + Bx^2 e^{ax} \cos(Bx) \\ + Cx e^{ax} \sin(Bx) + Dx e^{ax} \cos(Bx) \\ + E e^{ax} \sin(Bx) + F e^{ax} \cos(Bx)$$

① "Ride" the polynomial
② Include sines & cosines

D) Additive Combinations
- Treat as individual elements

$$\text{ie. } g(x) = x^2 + \sin(x) + e^x \cos(2x)$$

$$\text{guess } \Rightarrow Ax^2 + Bx + C + D \sin(x) + E \cos(x) \\ + F e^x \sin(2x) + G e^x \cos(2x)$$