Lecture Notes

Section 4.5a Part
Method of Undetermined Coefficients

I) We have explored the homogeneous D.E

$$ay'' + by' + cy = 0$$

⇒ We now consider the non-homogeneous form of the equation:

$$ay'' + by' + cy = g(x)$$

We can solve this equation if

$$g(x) = \begin{cases} \text{① polynomial} \\ \text{② exponential} \\ \text{③ } \sin/\cos \end{cases}$$

or any additive or multiplicative combination of the function.

II) The method of undetermined coefficients consists of 2 major steps:

① "guess" at a solution for the particular solution y_p → ② Execute a Solution Process

III) Today we focus on the Guess...

A) <u>Primary Rules</u>:

$g(x)$ is a polynomial, exponential, or sin/cos

1) i) $g(x) = 3x^2 - 2 \xrightarrow{\text{guess}} y_c = Ax^2 + Bx + C$

Polynomials

② 1st power term missing in $g(x)$ will not by missin in gues

① Start w/ highest power in $g(x)$ and form full polynomial

③ Values of coefficients do not affect guess

④ use $A, B, C, \ldots$ for unknown coefficient

⑤ Stop at 0^{th} power

ii) $g(x) = -100x^4 + 2x \xrightarrow{\text{Guess}} y_c = Ax^4 + Bx^3 + Cx^2 + Dx + E$

2) <u>Exponentials</u>

i) $g(x) = e^x \xrightarrow{\text{guess}} y_c = Ae^x$

ii) $g(x) = -10e^{3x} + 50e^{-6x} \xrightarrow{\text{guess}} y_c = Ae^{3x} + Be^{-6x}$

① Include all different exponential terms in gues

3) <u>Sin/Cos</u>

i) $g(x) = 10\sin(x) \xrightarrow{\text{guess}} y_c(x) = A\sin(x) + B\cos(x)$

① Must guess both sin & cos.

ii) $g(x) = 2\sin(x) - 4\cos(x) \xrightarrow{\text{guess}} y_c(x) = A\sin(x) + B\cos(x)$

① single guess covers both terms in $g(x)$

iii) $g(x) = \sin(x) + \cos(2x) \xrightarrow{\text{guess}} y_c(x) = A\sin(x) + B\cos(x) + C\sin(2x) + D\cos(2x)$

① THESE ARE DIFFERENT

B) <u>Secondary Rules</u> - Multiplicative combinations of two categories

1) polynomial * exponential

(i) $g(x) = x^2 e^{2x} \xrightarrow{\text{guess}} y_c = Ax^2e^x + Bxe^x + Cx$

① start w/ highest power in $g(x)$ and continue to 0^{th} power

(ii) $g(x) = (x^3+1)e^x \xrightarrow{\text{guess}} Ax^3e^x + Bx^2e^x + Cxe^x + De^x$

2) <u>polynomial * (sin/cos)</u>

(i) $g(x) = x^2\sin(x) \xrightarrow{\text{guess}} Ax^2\sin(x) + Bx^2\cos(x) + Cx\sin(x) + Dx\cos(x) + E\sin(x) + F\cos(x)$

① must "ride" highest power of polynomial to 0

② must include sin & cos in guess

3) exponential * (sin/cos)

(i) $-10e^x \sin(2x) \xrightarrow{\text{guess}} Ae^x \sin(2x) + Be^x \cos(2x)$

① must guess sines & cosines!

C) <u>Tertiary Rules</u> (multiplicative combination of all 3 elements)

i) $g(x) = 10x^2 e^{\alpha x} \sin(Bx) \xrightarrow{\text{guess}}$

$$Ax^2 e^{\alpha x} \sin(Bx) + Bx^2 e^{\alpha x} \cos(Bx)$$
$$+ Cx e^{\alpha x} \sin(Bx) + Dx e^{\alpha x} \cos(Bx)$$
$$+ E e^{\alpha x} \sin(Bx) + F e^{\alpha x} \cos(Bx)$$

① "Ride" the polynomial
② Include sines & cosines

D) <u>Additive Combinations</u>
- Treat as individual elements

i.e. $g(x) = x^2 + \sin(x) + e^x \cos(2x)$

guess $\Rightarrow$ $Ax^2 + Bx + C + D\sin(x) + E\cos(x)$
$+ Fe^x \sin(2x) + Ge^x \cos(2x)$