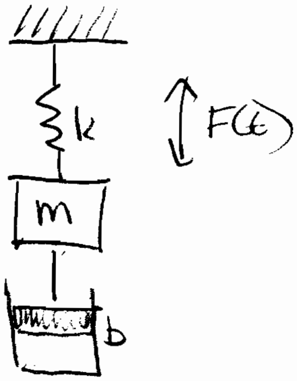
Lecture Notes

Section 5.1.3
SPRING/MASS SYSTEMS
DRIVEN MOTION



Assume that a external force is applied to the system (i.e. "shaking" the system)

I **Equation of motion:** $\boxed{mx'' + bx' + kx = F(t)}$

Note: This is merely the non-homogeneous form of the equation solved earlier

II **Example 6**

$m = 1/5$ slug

$k = 2$ lbs/ft

$b = 1.2$

$x(0) = 1/2$

$x'(0) = 0$

$f(t) = 5\cos(4t)$

$$\frac{1}{5}x'' + 1.2x' + 2x = 5\cos(4t)$$

$$x'' + 6x' + 10x = 25\cos(4t)$$

(A) Homogeneous Solution

$$(D^2 + 6D + 10)x = 0$$

$$\Rightarrow D = \frac{-6 \pm \sqrt{36-40}}{2} = \frac{-6 \pm \sqrt{-4}}{2}$$

$$= \frac{-6 \pm 2i}{2} = -3 \pm i$$

$$\Rightarrow x_H = c_1 e^{-3t}\cos(t) + c_2 e^{-3t}\sin(t)$$

(B) Particular Solution

$$X_p = A\sin(4t) + B\cos(4t)$$
$$X_p' = +4A\cos(4t) - 4B\sin(4t)$$
$$X_p'' = -16A\sin(4t) - 16B\cos(4t)$$

$$\Rightarrow (D^2 + 6D + 10)X_p = 5\cos(4t)$$

$$\Rightarrow (-16A - 24B + 10A)\sin(t)$$
$$(-16B + 24A + 10B)\cos(t) = 25\cos(4t)$$

$$\Rightarrow -6A - 24B = 0 \quad \times 4 \quad \Rightarrow \quad -24A - 96B = 0$$
$$24A - 6B = 25 \qquad\qquad \underline{24A - 6B = 25}$$
$$-102B = 25 \Rightarrow \boxed{B = \frac{-25}{102}}$$

$$\Rightarrow -6A - 24B = 0 \Rightarrow A = -4B \Rightarrow \boxed{A = 100/102}$$

$$\therefore \boxed{X_p = \frac{100}{102}\sin(4t) - \frac{25}{102}\cos(4t)}$$

$$X = c_1 e^{-3t}\cos(t) + c_2 e^{-3t}\sin(t) + \frac{50}{51}\sin(4t) - \frac{25}{102}\cos(4t)$$

$$X(0) = c_1 - \frac{25}{102} = \frac{1}{2} \Rightarrow c_1 = \frac{76}{102} = 38/51$$

$$X = \frac{38}{51}e^{-3t}\cos(t) + c_2 e^{-3t}\sin(t) + \frac{50}{51}\sin(4t) - \frac{25}{102}\cos(4t)$$
$$x' = -\frac{38}{17}e^{-3t}\cos(t) - \frac{38}{51}e^{-3t}\sin(t) - 3c_2 e^{-3t}\sin(t) + c_2 e^{-3t}\cos(t)$$
$$+ \frac{200}{51}\cos(4t) + \frac{100}{102}\sin(4t)$$
$$x'(0) = -\frac{38}{17} + c_2 + \frac{200}{51} = 0 \Rightarrow c_2 = -86/51$$

Transient
Dies w/time

$$\therefore X = \frac{38}{51}e^{-3t}\cos(t) - \frac{86}{51}e^{-3t}\sin(t)$$
$$+ \frac{50}{51}\sin(4t) - \frac{25}{102}\cos(4t)$$

Steady State

III) Shortcut For Annihilators - "Guess" Particular Solution

Let $C_n x^{(n)} + C_{n-1} x^{(n-1)} \dots C_2 x'' + c_1 x' + c_0 = f(t)$

If $f(t) = \_\_\_ \longrightarrow$ "Guess" $x_p = \_\_\_$

$$C_n x^n + C_{n-1} x^{n-1} \dots C_2 x^2 + C_1 x + C_0 \longrightarrow A x^n + B x^{n-1} \dots Q x + R$$

$$c_1 e^{at} \longrightarrow A e^{at}$$

$$\left.\begin{matrix} c_1 \cos(\omega t) \\ c_2 \sin(\omega t) \end{matrix}\right\} \longrightarrow A\cos(\omega t) + B\sin(\omega t)$$

$$\left.\begin{matrix} c_1 e^{at} \cos(\omega t) \\ c_2 e^{at} \sin(\omega t) \end{matrix}\right\} \longrightarrow A e^{at} \cos(\omega t) + B e^{at} \sin(\omega t)$$

$$\left.\begin{matrix} c_1 t \cos(\omega t) \\ c_2 t \sin(\omega t) \end{matrix}\right\} \longrightarrow \begin{matrix} At\cos(\omega t) + Bt\sin(\omega t) \\ + C\sin(\omega t) + D\cos(\omega t) \end{matrix}$$

This Method Actually Has a Name

"The Method Of Undetermined Coefficients"