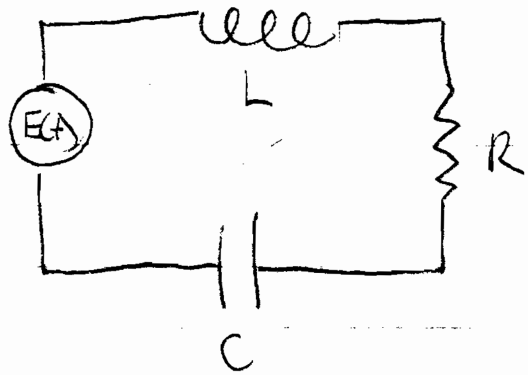
Lecture Notes

Section 5.1.4
Series Circuit Analogue

I KIRKOFFS Law



$$E(t) - L\frac{di}{dt} - Ri - \frac{1}{C}q = 0$$

voltage drops

$$\Rightarrow \quad L\frac{di}{dt} + Ri + \frac{1}{C}q = E(t)$$

$\Rightarrow$ but $\quad i = \dfrac{dq}{dt}$

$$\Rightarrow \quad \boxed{L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t)}$$

$L > 0$
$R > 0$
$C > 0$

II Solution:

$$D = \frac{-R \pm \sqrt{R^2 - 4L/C}}{2L}$$

if $R^2 - 4L/C$ $\begin{cases} < 0 & \text{underdamped} \\ = 0 & \text{critically damped} \\ > 0 & \text{over damped} \end{cases}$

III. Comparison to Vibration Problem

$$\begin{array}{ccccccccc}
L & q'' & + & R q' & + & \frac{1}{C} q & = & E(t) \\
\updownarrow & \updownarrow & & \updownarrow & & \updownarrow & & \updownarrow \\
m & x'' & + & b x' & + & k x & = & F(t)
\end{array}$$

i.e. this is the same problem mathematically, the application has changed, but the solution techniques are the same.