Lecture Notes

Section 8.1
Systems of Linear 1st ORDER EQUATIONS

I Systems of 1st ORDER DIFFERENTIAL EQNs

$$\frac{dx}{dt} = 3x - 4y$$

$$\frac{dy}{dt} = 4x - 7y$$

Note: That 'y' is included in '$\frac{dx}{dt}$' and 'x' is included in '$\frac{dy}{dt}$'

A. REWRITTEN IN MATRIX FORM

Let $X = \begin{bmatrix} x \\ y \end{bmatrix} \Longrightarrow X' = \frac{dX}{dt} = \begin{bmatrix} dx/dt \\ dy/dt \end{bmatrix}$

"big X"

$$\Rightarrow \begin{bmatrix} dx/dt \\ dy/dt \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 4 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow \boxed{X' = \begin{bmatrix} 3 & -4 \\ 4 & -7 \end{bmatrix} X}$$

B. CHECKING SOLUTION TO SYSTEM

is $\vec{X}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-5t} = \begin{bmatrix} e^{-5t} \\ 2e^{-5t} \end{bmatrix}$

a solution to the system above.

CHECK

COMPARE ✓✓

$$X_1' = \frac{d}{dt}\begin{bmatrix} e^{-5t} \\ 2e^{-5t} \end{bmatrix} = \begin{bmatrix} -5e^{-5t} \\ -10e^{-5t} \end{bmatrix}$$

$$\text{but } X_1' = \begin{bmatrix} 3 & -4 \\ 4 & -7 \end{bmatrix} \underbrace{\begin{bmatrix} e^{-5t} \\ 2e^{-5t} \end{bmatrix}}_{X_1} = \begin{bmatrix} 3e^{-5t} - 8e^{-5t} \\ 4e^{-5t} - 14e^{-5t} \end{bmatrix} = \begin{bmatrix} -5e^{-5t} \\ -10e^{-5t} \end{bmatrix}$$

C. Verify that a Solution Set is Fundamental

$X_2 = \begin{bmatrix} 2e^{t} \\ e^{t} \end{bmatrix}$ is also a solution to the system (can you verify this?)

X_1 & X_2 are a fundamental set if

$$\det\left(\begin{bmatrix} e^{-5t} & 2e^{t} \\ 2e^{-5t} & e^{t} \end{bmatrix}\right) \neq 0$$

$$\Rightarrow e^{-4t} - 4e^{-4t} = -3e^{-4t} \neq 0 \quad ✓✓$$

D) SUPERPOSITION PRINCIPLE

Since X_1 & X_2 for an independent or fundamental set than

$c_1 X_1 + c_2 X_2$ is also a solution

i.e. $X = c_1 \begin{bmatrix} e^{-5t} \\ 2e^{-5t} \end{bmatrix} + c_2 \begin{bmatrix} 2e^{t} \\ e^{t} \end{bmatrix}$

E) Verifying Particular Solutions

Consider the following non-homogeneous system of D.E.'s

$$X' = \underbrace{\begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}}_{\text{Homogeneous Part of Eqn}} X + \underbrace{\begin{pmatrix} 12t - 11 \\ -3 \end{pmatrix}}_{\text{Non-homogeneous Part of Equation}}$$

$\Rightarrow$ Show that $X_p = \begin{bmatrix} 3t - 4 \\ -5t + 6 \end{bmatrix}$ is the particular solution to the system

$$\Rightarrow \quad X_p' = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$$

COMPARE

bet $X_p' = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix} X_p + \begin{bmatrix} 12t-11 \\ -3 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 3t-4 \\ -5t+6 \end{bmatrix} + \begin{bmatrix} 12t-11 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{3t}-4-\cancel{15t}+18+\cancel{12t}-11 \\ \cancel{15t}-20-\cancel{15t}+18-3 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$$