

Lecture Notes

Section 9.4

Euler's Method for System's of DE's

I Theory

Recall from differential calculus

(a) $\lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

(b) since Δt can't be equal to zero we have

$$\left[\frac{\Delta x}{\Delta t} \approx \frac{dx}{dt} \right] \uparrow \text{approximately}$$

(c) the smaller Δt is, the better the approximation

II Euler's Method for Systems (Example)

Given $\left\{ \begin{array}{l} \frac{dx}{dt} = 2x + 4y \\ \frac{dy}{dt} = -x + 6y \end{array} \right.$

$\left. \begin{array}{l} \\ \end{array} \right\} \frac{dy}{dt} = -x + 6y$

$x(0) = -5, y(0) = 0$

Since: $\frac{\Delta x}{\Delta t} \approx \frac{dx}{dt}$ & $\frac{\Delta y}{\Delta t} \approx \frac{dy}{dt}$

Then: $\Delta x \approx \frac{dx}{dt} \Delta t$ & $\Delta y \approx \frac{dy}{dt} \Delta t$

If I know x, y, t and select a Δt
Then I can find Δx , & Δy

Example: Say I want to approximate

$x(5)$ and $y(5)$ using a step size $\Delta t = 1$

Set up a table:

t	x	y	$\frac{dx}{dt}$	$\frac{dy}{dt}$	Δt	Δx	Δy
Initial condition							
NEXT STEP $t = t + \Delta t$ $x = x + \Delta x$ $y = y + \Delta y$							

equations given

you pick Δt
smaller is better

$\approx \frac{dx}{dt} \Delta t$

$x \frac{dy}{dt} \Delta t$

i.e.

t	x	y	$\frac{dx}{dt}$	$\frac{dy}{dt}$	Δt	Δx	Δy
0	-0.5	0	-1.0	.5	.1	-.1	.05
1	-0.6	.05	-1.0	.9	.1	-.1	.09
2	-0.7	.14	-0.84	1.54	.1	-.08	.15
3	-0.78	.29	-0.39	2.55	.1	-.04	.25
4	-0.82	.55	.55	4.12	.1	.05	.41
5	-0.77	.96					

$x(5) \approx -0.77, y(5) = 0.96$

II Eulers Method For Higher ORDER DEs

i.e. $y'' + xy' + y = 0$ $y(0) = 1$, $y'(0) = 2$

let $y = y_1$

$$y' = \boxed{y_1 = y_2}$$

now rewrite original DE

$$y'' + xy' + y = 0$$

$$y'_2 + xy_2 + y_1 = 0 \Rightarrow \boxed{y'_2 = -xy_2 - y_1}$$

Now we have a system of equations:

$$y'_1 = y_2 \quad | \quad y_1(0) = 1$$

$$y'_2 = -xy_2 - y_1 \quad | \quad y_2(0) = 2$$

Note:

$$y'_1 = dy_1/dx$$

$$y'_2 = dy_2/dx$$

Use this to approximate, $y(1.2)$. $\Delta x = 0.1$

x	y_1	y_2	dy_1/dt	dy_2/dt	Δx	Δy_1	Δy_2
0	1	2	2	-1	0.1	0.2	-0.1
0.1	1.2	1.9	1.9	-1.32	0.1	0.19	-0.13
0.2	1.39	1.77					

$$y_1(0.2) \approx 1.39$$

$$y_2(0.2) = 1.77$$

III More Practice Setting Up Equations

$$x''' + 6x'' + 3x' + x = \sin(t) \quad \boxed{\begin{array}{l} \text{3rd Order} \\ \text{D.E.} \end{array}}$$

$$\begin{aligned} \text{let } x &= x_1 \\ x' &= x'_1 = x_2 \\ x'' &= x''_1 = x'_2 = x_3 \end{aligned} \quad \leftarrow \text{stop at } x_3$$

$$\text{rewrite } x''' + tx'' + 3x' + x = \sin(t)$$

$$\therefore x_3' = \sin(t) - tx_3 - 3x_2 - x_1$$

1

EXPRESS IN
TERMS OF
 x_i using
lowest possible
derivative

$$\begin{aligned}x_1' &= x_2 \\x_2' &= x_3 \\x_3' &= \sin(t) - tx_3 - 3x_2 - x_1\end{aligned}$$

Note:

- For an n^{th} order DE we need to use $\{x_1, x_2, \dots, x_n\}$
 - A system of n 1st order DEs is generated

Resulting Euler's Table

t	x_1	x_2	x_3	$\frac{dx_1}{dt}$	$\frac{dx_2}{dt}$	$\frac{dx_3}{dt}$	Δt	Δx_1	Δx_2	Δx_3
-----	-------	-------	-------	-------------------	-------------------	-------------------	------------	--------------	--------------	------------------------------------