Lecture Notes

Section 11.3
Sine & Cosine Series

I Odd & Even Functions

A. Even Function

(1) Algebraic Definition

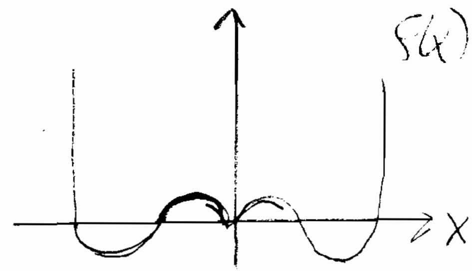
$$\boxed{f(-x) = f(x)}$$

(2) Examples

$f(x) = x^2 \Rightarrow f(-x) = (-x)^2 = x^2 = f(x)$

$f(x) = \cos(x) \Rightarrow f(-x) = \cos(-x) = \cos(x) = f(x)$

(3) Plot



"reflects through y-axis"

B. Odd Functions

(1) Algebraic Definition

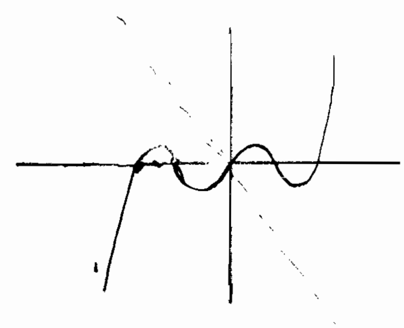
$$\boxed{f(-x) = -f(x)}$$

(2) Examples

$f(x) = x^3 \Rightarrow f(-x) = (-x)^3 = -x^3 = -f(x)$

$f(x) = \sin(x) \Rightarrow f(-x) = \sin(-x) = -\sin(x) = -f(x)$

(3) Plot



"reflects through line $y=-x$"

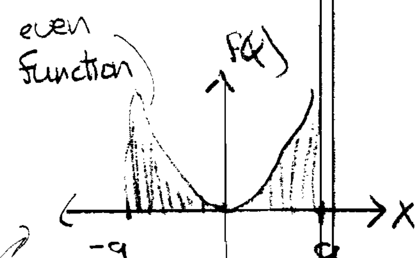
II Properties of Even/Odd Functions

A.

$f(x)$ &	$g(x)$	$\Longrightarrow$ $f(x)g(x)$	$f(x)+g(x)$
odd	odd	even	odd
odd	even	odd	neither
even	odd	odd	neither
even	even	even	even

B. If $f(x)$ is even then

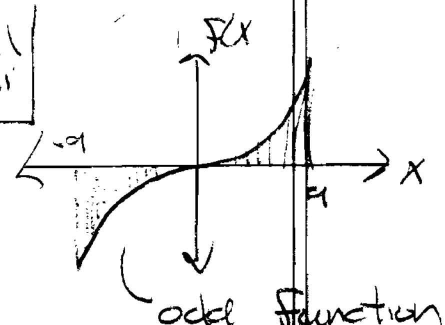
$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$$



"graphical proofs"

C. If $f(x)$ is odd then

$$\int_{-a}^{a} f(x)\,dx = 0$$



III Fourier Sin Series for Odd Functions

Assume $f(x)$ is odd on interval $[-p,p]$

Then
$$\boxed{\begin{aligned} f(x) &\sim \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{p} \\ b_n &= \frac{2}{\pi}\int_0^p f(x)\sin\left(\frac{n\pi x}{p}\right)dx \end{aligned}}$$

There are <u>no</u> cosine components because $\cos(x)$ is an <u>even</u> function

IV. Fourier Cosine Series for an Even Function

Assume $f(x)$ is even on $[-p, p]$

Then $f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{p}\right)$

$$\boxed{\begin{aligned} a_0 &= \frac{2}{p}\int_0^p f(x)\,dx \\ a_n &= \frac{2}{p}\int_0^p f(x)\cos\left(\frac{n\pi x}{p}\right) \end{aligned}}$$

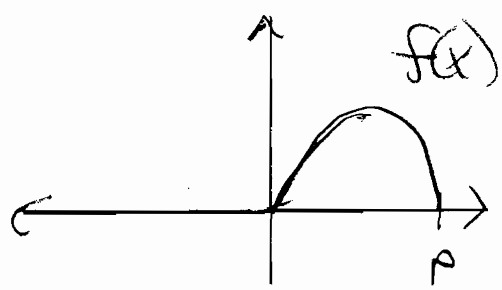
There are no sine components because $\sin(x)$ is an even function

V. Example 2 (410)

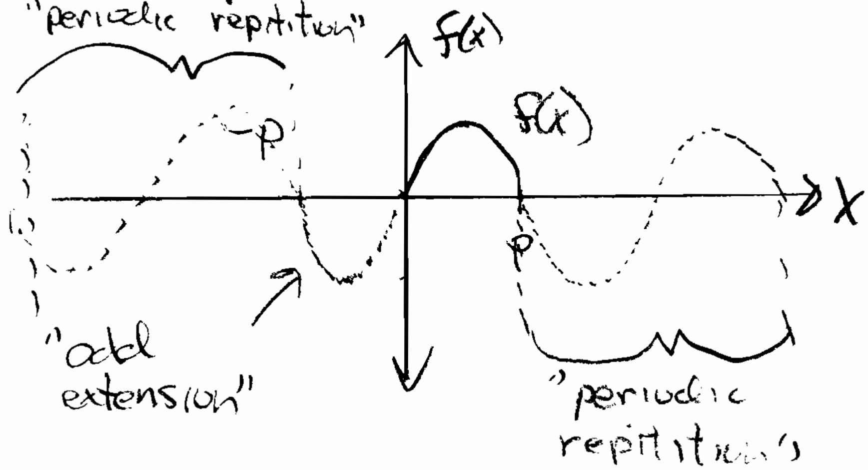
VI. Half Range Expansions

Assume $f(x)$ is defined on $[0, p]$

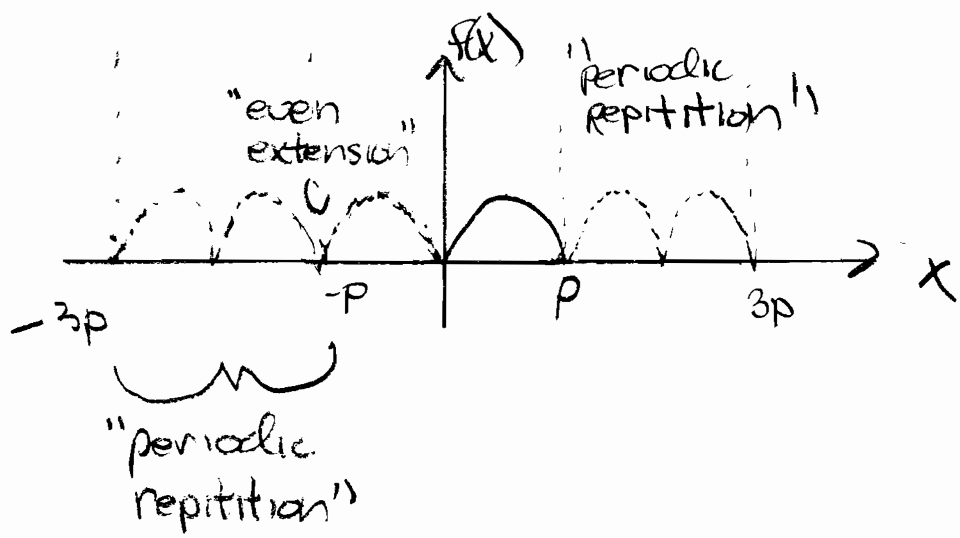
I can write $f(x)$ is either a Fourier sine or a Fourier cosine expansion.



A. If I write as a sine series, I will get an odd extension



B. If I write a cosine series I get an even extension



C. Example 3 (412)