Lecture Notes

Section 6.1.1a
REVIEW OF POWER Series

I **Power Series "centered on a"**

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots$$

Example $\therefore$

$$\sum_{k=0}^{\infty} \frac{1}{2^k}(x-1)^k = 1 + \frac{1}{2}(x-1) + \frac{1}{4}(x-1)^2 + \dots$$

A. Power Series Represents Function IF THEY ARE CONVERGENT i.e. THE ABOVE FUNCTION IS CONVERGENT FOR $-2 < x < 2$

** $\boxed{\text{Go 1}}$

matlab Demo

C **Ratio TEST FOR Convergence**

$$|(x-a)| \lim_{n\to\infty} \left|\frac{c_{n+1}}{c_n}\right| < 1$$

Example $\therefore$

$$|(x-1)| \lim_{n\to\infty} \left| \frac{1}{2^{n+1}} \frac{2^n}{1} \right| < 1$$

"Radius of Convergence"

$$\Rightarrow |x-1| \cdot \frac{1}{2} < 1 \Rightarrow |x-1| < 2$$

$$\Rightarrow -2 < x-1 < 2 \Rightarrow -1 < x < 3$$

Interval of convergence

II)

Shifting Indices

<u>Example:</u> (a) Say I have a power series

$$\sum_{n=2}^{\infty} (n)(n-1)C_n x^{n-2}$$

(b) I wish to express it in terms of x^k

$$\text{let}\begin{cases} k = n-2 \\ \Rightarrow n = k+2 \\ \Rightarrow n-1 = k+1 \\ \Rightarrow n=2 \Rightarrow k=0 \end{cases}$$

(c) Rewrite (a)

$$\sum_{k=0}^{\infty} (k+2)(k+1)C_{k+2}\, x^k$$

(d)

<u>Note:</u>

$$\sum_{n=2}^{\infty} (n)(n-1)C_n x^{n-2} = (2)(1)C_2 x^0 + (3)(2)C_1 x^1 \dots$$
$$= 2C_2 + 6C_1 x + \dots$$

$$\sum_{k=0}^{\infty} (k+2)(k+1)C_{k+2}\, x^k = (2)(1)C_2 x^0 + (3)(2)C_1 x$$
$$= 2C_2 + 6C_1 x \quad \dots,$$

<u>The series did not change!!</u>

III Why We Shift Indices

Example 1 (222) Adding 2 Power Series

① $$\sum_{n=2}^{\infty} (n)(n-1)c_n x^{n-2} + \sum_{n=0}^{\infty} c_n x^{n+1}$$

Express Both in terms of x^k

$\begin{cases} k = n-2 \\ \Rightarrow n = k+2 \\ \Rightarrow n=2 \rightarrow k=0 \end{cases}$ $\begin{cases} k = n+1 \\ \Rightarrow n = k-1 \\ \Rightarrow n=0 \rightarrow k=1 \end{cases}$

② $$\sum_{k=0}^{\infty} (k+2)(k+1) c_{k+2} x^k + \sum_{k=1}^{\infty} c_{k-1} x^k$$

strip of 1st term so both series start a $k=1$

$$\Rightarrow (2)(1)c_2 x^0 + \underbrace{\sum_{k=1}^{\infty} (k+2)(k+1) c_{k+2} x^k + \sum_{k=1}^{\infty} c_{k-1} x^k}_{\text{combine}}$$

"0-term"

③ $$\boxed{2c_2 + \sum_{k=1}^{\infty} \left[(k+2)(k+1)c_{k+2} + c_{k-1}\right] x^k}$$