Lecture Notes

Section 6.1
Power Series

Example 3, P. 225

(I) $y'' + xy = 0$

$$y = \sum_{n=0}^{\infty} c_n x^n \Rightarrow xy = \sum_{n=0}^{\infty} c_n x^{n+1}$$

$$y' = \sum_{n=1}^{\infty} c_n(n)x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} c_n(n)(n-1)x^{n-2}$$

$$\Rightarrow \sum_{n=2}^{\infty} c_n(n)(n-1)x^{n-2} + \sum_{n=0}^{\infty} c_n x^{n+1} = 0$$

II Indice Shift to make all powers equal x^k

$\uparrow$

$k = n-2$ | $k = n+1$
$n = k+2$ | $n = k-1$
$n=2 \Rightarrow k=0$ | $n=0 \Rightarrow k=1$

$$\sum_{k=0}^{\infty} c_{k+2}(k+2)(k+1)x^k + \sum_{k=1}^{\infty} c_{k-1}x^k$$

III peel off zero-terms

$$C_2(2)(1)x^0 + \sum_{k=1}^{\infty} C_{k+2}(k+1)(k+2)x^k + \sum_{n=1}^{\infty} C_{k-1}x^k = 0$$

combine

$$2C_2 + \sum_{k=1}^{\infty}\left[C_{k+2}(k+1)(k+2) + C_{k-1}\right]x^k = 0$$

$$\Rightarrow 2C_2 = 0 \Rightarrow \boxed{C_2 = 0} \Rightarrow k=0$$

$$\Rightarrow C_{k+2}(k+1)(k+2) + C_{k-1} = 0$$

$$\Rightarrow \boxed{C_{k+2} = \frac{-C_{k-1}}{(k+1)(k+2)}}$$

III <u>List Recurrance Relations For 1ST 9 or 10 Terms</u>

$k=0 \Rightarrow C_2 = 0$

$k=1 \Rightarrow C_3 = \frac{-C_0}{(2)(3)}$

Note: It Appears that C_1 & C_0 are independent

$k=2 \Rightarrow C_4 = \frac{-C_1}{(3)(4)}$

$k=3 \Rightarrow C_5 = \frac{-C_2}{(4)(5)} = 0$

$k=4 \Rightarrow C_6 = \frac{-C_3}{(5)(6)} = \frac{-1}{(5)(6)}\frac{-C_0}{(2)(3)} = \frac{C_0}{(2)(3)(5)(6)}$

$$k=5 \Rightarrow C_7 = \frac{-C_4}{(6)(7)} = \frac{-1}{(6)(7)}\,\frac{-C_1}{(3)(4)} = \frac{C_1}{(3)(4)(6)(7)}$$

$$k=6 \Rightarrow C_8 = \frac{-C_5}{(7)(8)} = 0$$

$$k=7 \Rightarrow C_9 = -\frac{C_6}{(8)(9)} = \frac{-C_0}{(2)(3)(5)(6)(8)(9)}$$

$$k=8 \Rightarrow C_{10} = \frac{-C_7}{(9)(10)} = \frac{-C_1}{(3)(4)(6)(7)(9)(10)}, \quad \text{ETC}$$

V <u>WRITE SOLUTIONS IN TERMS OF C_0 & C_1</u>

$$\Rightarrow y_1 = C_0\Big[\underbrace{1}_{C_0 \Leftrightarrow x^0} - \underbrace{\frac{1}{(2)(3)}}_{C_3 \Leftrightarrow x^3} x^3 + \underbrace{\frac{1}{(2)(3)(5)(6)}}_{C_6 \Leftrightarrow x^6} x^6 - \underbrace{\frac{1}{(2)(3)(5)(6)(8)(9)}}_{C_9 \Leftrightarrow x^9} x^9 \cdots\Big]$$

$$y_2 = C_1\Big[x - \underbrace{\frac{1}{(3)(4)}}_{C_4} x^4 + \underbrace{\frac{1}{(3)(4)(6)(7)}}_{C_7} x^7 - \underbrace{\frac{1}{(2)(3)(5)(6)(8)(9)}}_{C_{10}} x^{10} \cdots\Big]$$

$$\Downarrow$$

$$\Rightarrow \boxed{y = C_0 y_1 + C_1 y_2}$$

<u>Side Note</u>:
Solution Called Airy's
Equation.