

Time Dependent vs. Time Independent Schrödinger Equation

So far we have used the time independent Schrödinger equation to look at the various classical problems in quantum mechanics. In one dimensional rectangular coordinates the time independent form is:

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x).$$

Where:

| | |
|---------|---|
| \hbar | $\frac{h}{2\pi}$ (reduced Planck's constant) |
| h | Planck's constant (describes size of quanta in quantum mechanics) |
| m | mass of particle |
| ψ | wave function (replaces the concept of trajectory in classical mechanics) |
| $V(x)$ | potential energy of particle |
| E | total energy of particle |

The general form of the Schrödinger equation includes a time element (also known as the time dependent Schrödinger equation):

$$i\hbar \frac{\partial\Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t)$$

Homework:

- Using separation of variables and the time dependent Schrödinger equation derive the time independent form of the Schrödinger equation. (Hint: (a) Let $\Psi(x,t) = T(t)\psi(x)$ (b) When you have separated the variables set the equations equal to E vice $-\lambda$.)
- Solve for $T(t)$.