**The Hermite Differential Equation 🡪**

**Express DE as a Power Series**

This is a homogeneous 2nd order differential equation with non-constant coefficients. Typically m is a non-negative integer. We will solve this using power series technique. Assume the solution to the differential equation:

Therefore the differential equation can be rewritten as:

I can move the in the center term to the inside of the summation:.

**Shift Indices so all terms are in the form**

Note that is the same as . (Why is this true?). This clever little trick gets me out of the step in which I “strip terms”. With this in mind I get:

Now combine the terms as follows:

From this we conclude:

Therefore:

**Apply Initial Conditions to Solve for Constants**

Given the initial conditions y(0)=a, and y’(0)=b, the values for and can be obtained as follows:

**Hermite Polynomials of Even Order**

Now consider the following initial conditions:

Here is a double factorial term defined as follows:

For example: 7!!=(7)(5)(3)(1)=105.

From the recursion relationship above, we see that if then all when k is odd. Now we find , for even values of k.

For purposes of example, let m be an even integer, i.e. **m=6**. Then:

Therefore:

We test the solution by putting it back into the Hermite DE for m=6, i.e.. The first and second derivatives of y are:

Substituting this into the DE yields:

= (-768+768)) √√√

The solution above is called a Hermite polynomial of **order 6** and is denoted by Note that any multiple of this polynomial is also considered a Hermite polynomial of order 6. Hermite polynomials of other even valued orders can be obtained by using the same initial conditions and varying the values of m over the even numbers.

**Hermite Polynomials of Odd Order**

In order to obtain Hermite polynomials of odd order we specify the following initial conditions:

),

which implies that and . Then specify m to be any odd number, i.e. if **m=5** then:

Therefore:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  | | --- | --- | --- | | **A Summary of the First Ten Hermite Polynomials** | | | | **Initial Conditions** |  |  | | **Even Polynomials** | 0 |  | | 2 |  | | 4 |  | | 6 |  | | 8 |  | | **Odd Polynomials** | 1 |  | | 3 |  | | 5 |  | | 7 |  | | 9 |  | |

**Orthogonality Property of Hermite Polynomials**

A family of functions is said to be orthogonal with respect to a weight over an interval if the following is true:

ermite polynomials form an orthogonal set of functions for the weight over the interval . The exact relation is:

This will not be proved, but can the demonstrated using any of the Hermite polynomials listed in the table. The property of orthogonality becomes important when solving the Harmonic oscillator problem.

**Homework**

1. Following the example for deriving **,** derive **.**
2. Verify by substituting it into the Hermite differential equation .
3. Calculate
   1. Directly using your TI200.
   2. Indirectly using the Orthogonality Property above.
4. Calculate
   1. Directly using your TI200.
   2. Indirectly using the Orthogonality Property above.

