

Time Independent Schrödinger Equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = E\Psi$$

Coordinate System		Application	LaPlacian Operator ∇^2	Potential Energy V	Total Energy (decrete) E (# of eigenvalues)
Rectangular $\Psi(x, y, z), V(x, y, z)$	1-D	Particle in a Box	$\frac{d^2}{dx^2}$	Infinite Well (i.e. $V = 0$ or $V = \infty$)	1
		Harmonic Oscillator		$\frac{1}{2}M\omega^2x^2$	1
	2-D	Particle in a Box	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$	Infinite Well	2
	3-D	Particle in a Box	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$	Infinite Well	3
Polar $\Psi(r, \theta), V(r, \theta)$	1-D ($R = r$)	Particle on a Ring	$\frac{1}{R^2} \frac{\partial^2}{\partial \theta^2}$	Infinite Well	1
	2-D	Particle on a Disc	$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$	Infinite Well	2
Spherical $\Psi(\rho, \theta, \phi), V(\rho, \theta, \phi)$	2-D ($\rho = R$)	Rigid Rotor	$\frac{1}{R^2 \sin(\phi)} \frac{\partial}{\partial \phi} \left(\sin(\phi) \frac{\partial}{\partial \phi} \right) + \frac{1}{R^2 \sin^2(\phi)} \frac{\partial^2}{\partial \theta^2}$	Infinite Well	2
	3-D	Hydrogen Atom	$\frac{1}{\rho^2} \frac{\partial}{\partial \rho} \left(\rho^2 \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2 \sin(\phi)} \frac{\partial}{\partial \phi} \left(\sin(\phi) \frac{\partial}{\partial \phi} \right) + \frac{1}{\rho^2 \sin^2(\phi)} \frac{\partial^2}{\partial \theta^2}$	$-\frac{1}{4\pi\epsilon_0} \frac{Ze^2}{\rho}$	3

Other Solvable Systems

- (see http://en.wikipedia.org/wiki/List_of_quantum-mechanical_systems_with_analytical_solutions)
- The [free particle](#), [delta potential](#), [finite potential well](#), [One-dimensional triangular potential](#), [particle in a spherically symmetric potential](#), [particle in a one-dimensional lattice \(periodic potential\)](#), [Morse potential](#), [step potential](#), [symmetric top](#), [Hooke's atom](#), [Spherium](#)