

Score:

Name:

Solutions

Period (circle one): 1 2 3 4 5 6

Team (circle one): a b c d e f

SM286 – Quiz 4 – Section 3.1 Newton's Law of Cooling

1. A thermometer is removed from a room where the temperature is 70°F and is taken outside, where the air temperature is 10°F. After half a minute the thermometer reads 50°F. How long will it take for the thermometer to reach 15°F

$$T(0) = 70^\circ\text{F}$$

$$T_m = 10^\circ\text{F}$$

$$T(30) = 50^\circ\text{F}$$

↑
sec

$$\frac{dT}{dt} = k(T - T_m)$$

$$\Rightarrow \int \frac{dT}{(T-10)} = \int k dt \Rightarrow \ln(T-10) = kt + C$$

$$\Rightarrow T - 10 = e^{kt+C} = ae^{kt}$$

$$\Rightarrow T = ae^{kt} + 10 \Rightarrow T(0) = a + 10 = 70 \Rightarrow a = 60$$

$$\Rightarrow T = 60e^{kt} + 10 \Rightarrow T(30) = 60e^{30k} + 10 = 50$$

$$\Rightarrow e^{30k} = \frac{40}{60} = \frac{2}{3} \Rightarrow k = \frac{1}{30} \ln\left(\frac{2}{3}\right) \approx -0.0135$$

$$\Rightarrow T = 60e^{-0.0135t} + 10$$

$$T(?) = 60e^{-0.0135t} + 10 = 15$$

$$\Rightarrow e^{-0.0135t} = \frac{5}{60} = \frac{1}{12} \Rightarrow 184.1 \text{ sec}$$