Score:

Name: Solutions
Period (circle one): 1 2 3 4 5 6
Team (circle one): a b c d e f

SM286 – Quiz 7 – Section 4.3b
Homogeneous DEs with Constant Coefficients (Complex Roots)

1. Solve the following initial value problem (do not combine sin/cos terms):

$$\frac{d^2x}{dt^2} + k^2x = 0, \quad x(0) = a, \quad x'(0) = b$$

$(D^2+k^2)x = 0 \Rightarrow D = \pm ki \Rightarrow x = c_1\sin(kt) + c_2\cos(kt)$

$\Rightarrow x(0) = c_2 = a \Rightarrow x = c_1\sin(kt) + a\cos(kt)$

$\Rightarrow x'(t) = kc_1\cos(kt) - ka\sin(kt)$

$x'(0) = kc_1 = b \Rightarrow c_1 = \frac{b}{k}$

$\therefore$ $\boxed{x = \frac{b}{k}\sin(kt) + a\cos(kt)}$

2. What is the general solution for (let t be the independent variable:

$$(D^2 + 3D + 2)(D^2 - 1)(D^2 + 1)(D^2 + 4D + 5)y = 0$$

$(D+1)(D+2)(D+1)(D-1)(D^2+1)(D^2+4D+5)y = 0$

REPEAT

$D = -1, -2, -1, 1, \pm i, -2 \pm i$

$\Rightarrow$ $\boxed{y = c_1e^{-t} + c_2te^{-t} + c_3e^{-2t} + c_4e^{t} + c_5\sin(t) + c_6\cos(t) + c_7e^{-2t}\sin(t) + c_8e^{-2t}\cos(t)}$