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SM286 – Quiz 10 – Section 4.5b
Solving DEs by Undetermined Coefficients

1. Solve $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = e^{-t}$, $x(0) = 1$, $x'(0) = -1$

① $(D^2 + 3D + 2)x = 0 \Rightarrow (D+1)(D+2)x = 0$ $D = -1, -2$

$$x_H = c_1 e^{-t} + c_2 e^{-2t}$$

② $x_p = A e^{-t}$ ③ decompose \Rightarrow

$$x_p = A t e^{-t}$$

$$x_p' = A e^{-t} - A t e^{-t}$$

④ $x_p'' + 3x_p' + 2x_p = (-2 + 3)A e^{-t}$ $x_p'' = -2A e^{-t} + A t e^{-t}$

$$= A e^{-t} = e^{-t} \quad (\text{note } A t e^{-t} \text{ terms canceled out})$$

$$\Rightarrow \boxed{A=1}$$

⑤ $x = c_1 e^{-t} + c_2 e^{-2t} + t e^{-t}$
 $x' = -c_1 e^{-t} - 2c_2 e^{-2t} + e^{-t} - t e^{-t}$

⑥ $x(0) = c_1 + c_2 = 1$

$$x'(0) = -c_1 - 2c_2 + 1 = -1$$

$$-c_2 + 1 = 0 \Rightarrow \boxed{c_2 = 1} \Rightarrow c_1 = 0$$

$$\boxed{x = e^{-2t} + t e^{-t}}$$