

Score:

Name: Solutions
Period (circle one): 1 2 3 4 5 6
Team (circle one): a b c d e f

SM286 – Quiz 13 – Appendix 2
Determinants/Inverses/Eigenvalues/Eigenvectors

1. Let $A = \begin{bmatrix} 3 & 4 \\ 3 & 2 \end{bmatrix}$. Find:

a. Det(A)

$$(3)(2) - (3)(4) = \boxed{-6}$$

b. A^{-1} (Shortcut: Cramer's Rule ☺)

$$\frac{1}{\det(A)} \begin{bmatrix} 2 & -4 \\ -3 & 3 \end{bmatrix} = \frac{-1}{6} \begin{bmatrix} 2 & -4 \\ -3 & 3 \end{bmatrix} = \boxed{\begin{bmatrix} -1/3 & 2/3 \\ 1/2 & -1/2 \end{bmatrix}}$$

c. The Eigenvalues and Eigenvectors for A (express eigenvector elements as integers-sorry, no short cuts).

$$\det \begin{bmatrix} 3-\lambda & 4 \\ 3 & 2-\lambda \end{bmatrix} = 0 \Rightarrow (3-\lambda)(2-\lambda) - 12 = 0 \Rightarrow \lambda^2 - 5\lambda - 6 = 0$$

$$(\lambda+1)(\lambda-6) = 0 \quad \lambda = -1, 6$$

$$\boxed{\lambda = -1}$$

$$\begin{bmatrix} 4 & 4 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow 4x_1 + 4x_2 = 0 \Rightarrow x_1 = -x_2 \Rightarrow \vec{v}_1 = \boxed{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

$$\boxed{\lambda = 6}$$

$$\begin{bmatrix} -3 & 4 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow -3x_1 + 4x_2 = 0 \Rightarrow x_2 = \frac{3}{4}x_1 \Rightarrow \vec{v}_2 = \boxed{\begin{bmatrix} 4 \\ 3 \end{bmatrix}}$$